DRONACHARYA SR. SEC. SCHOOL MUBARIKPUR (JH)
AFFILIATED TO CBSE NEW DELHI, Aff No. 530945

Summary Holidays Homework

Class: - XII(Non.Medical)

Dear Students,

As the summer sun rises higher and the days stretch longer, it's time to pause, reflect, and recharge. The months of May and June bring with them not just warmth, but also the perfect opportunity to reconnect—with ourselves, with our families, and with the world around us.

Let us remember: "A mind stretched by new experiences can never go back to its old dimensions." This summer, let curiosity lead you, compassion guide you, and creativity energize you.

As part of the Summer Holiday Engagement 2025, we encourage you to stay indoors during peak hours, remain hydrated, and take thoughtful care of the ecology and environment around you. Even small steps—like reducing plastic use, saving electricity, and caring for plants—can bring meaningful change.

This year's holiday homework is designed to nurture Social, Emotional, and Experiential Learning. Whether you're experimenting in the kitchen, helping out at home, writing in your journal, or diving into a book—know that each activity contributes to your growth.

Let this summer be more than just a break. Let it be a season of purposeful rest, joyful discovery, and mindful action.S

I believe in your potential to make a difference and look forward to seeing your spirit shine through in every page of your work.

Stay safe, stay curious, and above all—stay kind.

Warm wishes,

Class Teacher

GENERAL INSTRUCTIONS

- **Complete your homework in the given time period.**
- **Make a separate single notebook for all subjects.**

English	. Reading and Writing 1. Newspaper Reading and Report Collection - Read the newspaper daily with a special emphasis on school-based reports. - Cut out at least 10 different newspaper reports and use them as a ready reference for report writing. Pay attention to the language used to get a fair idea of how reports must be written. 2. Letter Writing: Solve 6 letter writing assignments (3 of each type, letter to the editor & Application for job) from your BBC Compacta. II. Reading and Comprehension Solve 3 Unseen passages of TYPE 1, and 3 unseen passages of TYPE 2 from Module ONE in your BBC Compacta. Type 1- passages to assess comprehension, interpretation, inference and vocabulary Type 2- Case Based Factual Passage III. Notice and Summaries 1. Notice Writing - Solve any 5 Notice Writing Assignments form your BBC Compacta. 2. Summaries of Lessons - Write brief summaries of the following lessons in 250-300 words: - "The Last Lesson" - "Deep Water" - "Lost Spring" IV. Summaries, and Biographical Sketches 1. Summaries of Poems - Write brief summaries of the following poems in 120-150 words: - "My Mother at Sixty Six" - "A Thing of Beauty" - "Keeping Quiet" 2. Biographical Sketches - Write biographical sketches of the following poets:

	- Robert Frost - John Keats V. Listening and Speaking Tasks (Throughout the Vacation) Prepare a speech on any one of the given topics (in written format as well as be prepared to deliver it in class) - Importance of Elder Members in a Family (in reference to the poem "My Mother at Sixty Six") - Impact of Poverty and Societal Inequality on Childhood - Themes of Trust and Kindness - Prepare your points and be ready to discuss these topics with your classmates. VI. Solve the Extract based questions from Module 5 of your BBC Compacta for the following chapters and poems : 1. The Last Lesson 2. Lost Spring 3. Deep Water 4. My Mother at Sixty six 5. Keeping Quiet 6. A Thing of Beauty -General Instructions: - Maintain a separate notebook for your holiday homework. - Ensure that your work is neat and well-organized. - Be prepared to discuss and share your work with your classmates when school reopens .
Physics	**Concept Builder Notebook :-** Prepare handwritten summary notes (5-6 pages per chapter) covering the following: Key definitions and laws (e.g., Coulomb's Law, Gauss's Law, Ohm's Law, Biot-Savart Law) Important formulas with SI units. Diagrams with proper labeling. **Flashcard Revision Kit:-** Create a set of 25 flashcards to revise key concepts from the chapters: Chapters Covered: Electrostatics, Current Electricity, Magnetic Effect of Current, Magnetism Electromagnetic Induction (EMI) Alternating Current **Instructions** to Create Flashcards: **Material**: Use small index cards or cut A4 sheets into 4 equal parts. **Front Side**: Write a question, term, law, or formula name. **Back Side**: Write the answer, explanation, diagram, or derivation tip. **Mind Map Creation :-** Make a mind map on one of the following topics (A4 size, colored): Magnetic effect of current , Current Electricity, Alternating Current. Include: Definitions, Laws, Formulae, Diagrams, Practical examples. Use branches

	and arrows to show relationships. Add icons, keywords, and visual elements to make it appealing and easy to revise. **Electricity Consumption Audit:-** Make a table of at least 5 home appliances. For each list: Power rating , Hours used per day. Total energy consumed in a week. Calculate the total cost using Rs. 6/unit **Poster** – AC vs DC :- Make a visually engaging poster comparing Alternating Current (AC) and Direct Current (DC). Include: Definitions, Graphs of variation with time, Applications , Sources (batteries vs power supply) **" Revise whole syllabus for a test after summer Vacations"**
Chemistry	1. Handwritten Notes of All 4 Chapters ✍ Prepare notes chapter-wise in your Chemistry notebook: Write definitions, laws, and key formulas Include solved numericals where needed (especially for Solutions and Electrochemistry) Make reaction charts for Haloalkanes & d-block Draw important diagrams (like conductivity cell, galvanic cell, etc.) ? Use highlighters or colored pens to make key points stand out. ? 2. Make One Informative Chart or Model (Choose Any One) ? You may choose: Chart of types of solutions with examples Electrochemical series or cell diagram (Galvanic cell) Reactions of Haloalkanes in flowchart form Electronic configuration of d- and f-block with periodic table placement ? Use simple materials: cardboard, color pens, chart paper, etc. ? 3. Solve Practice Questions ? Solve the following in your notebook (mention question + full answer): 10 numericals from "Solutions" (based on molarity, molality, Raoult's law) 5 numericals from "Electrochemistry" 5 Intext/NCERT-based questions each from d & f block and Haloalkanes ? Total: 30 important board-style questions. ? 4. Prepare 10 Short Questions and Answers from Each Chapter ? Write in simple language, using your own words: These questions may be used for a quiz/test after vacation. **Class 12 th chemistry Assignment** Multiple Choice Questions (MCQs) 1. Which of the following will show maximum elevation in boiling point? A) 1 molal NaCl B) 1 molal urea C) 1 molal glucose D) 1 molal $BaCl_2$

2. The unit of molality is:
A) mol/L
B) mol/kg
C) mol/m^3
D) mole
3. Which of the following is not a colligative property?
A) Osmotic pressure
B) Depression in freezing point
C) Elevation in boiling point
D) Enthalpy of vaporization
4. Which law relates vapour pressure of a solution to the mole fraction of solvent?
A) Raoult's Law
B) Henry's Law
C) Dalton’s Law
D) Boyle’s Law
5. The molar conductivity of a strong electrolyte:
A) Decreases with dilution
B) Increases with dilution
C) Remains constant
D) First increases then decreases
6. Which of the following is used as a reference electrode?
A) Zinc electrode
B) Calomel electrode
C) Copper electrode
D) Silver electrode
7. Electrolysis of aqueous NaCl solution gives:
A) Na
B) NaOH and Cl_2
C) H_2 and NaOH
D) NaOH and O_2
8. Which of the following haloalkanes undergo SN1 reaction most readily?
A) CH_3Br
B) C_2H_5Br
C) $(CH_3)_3CBr$
D) $CH_3CH_2CH_2Br$

9. The best method to prepare alkyl fluorides is:
A) Halogenation
B) Finkelstein reaction
C) Swarts reaction
D) Kolbe’s reaction
10. What is the IUPAC name of $CH_3CHBrCH_2CH_3$?
A) 1-bromobutane
B) 2-bromobutane
C) 3-bromobutane
D) Butyl bromide

11. Define molality and mole fraction. Write their units.

12. State Henry's Law and its significance in soft drinks and scuba diving.

13. Calculate the mass of urea required to be dissolved in 500 g of water to lower the freezing point by 2 K. (K_f = 1.86 K kg mol^{-1})

14. Define the term 'abnormal molar mass' with reference to colligative properties.

15. What is the effect of temperature on conductivity and molar conductivity?

16. Differentiate between electrolytic cell and galvanic cell (any 3 points).

17. Define cell constant and write its SI unit.

18. Write the Nernst equation for a cell and explain the terms involved.

19. What is Kohlrausch's Law? State its two applications.

20. How is chlorobenzene prepared from aniline? Name the reaction involved.

21. What is Raoult's Law? Derive the expression for the total vapour pressure of a binary solution.

22. Define osmotic pressure. Derive an expression to relate osmotic pressure with molar mass of solute.

23. Derive the relationship between degree of dissociation (α) and van't Hoff factor (i). How does it affect colligative properties?

24. Describe the construction and working of a Daniel cell with appropriate reactions at anode and cathode.

25. Calculate the EMF of the following cell at 298 K:
$Zn(s) \mid Zn^{2+}$ (0.1 M) $\|$ Cu^{2+} (1 M) $\mid Cu(s)$
$E^\circ_{Zn^{2+}/Zn} = -0.76$ V, $E^\circ_{Cu^{2+}/Cu} = +0.34$ V

26. Explain how conductivity and molar conductivity vary with dilution for strong and weak electrolytes.
27. Explain SN1 and SN2 mechanisms with suitable examples.
28. Describe the Swarts reaction and Finkelstein reaction. Write balanced equations.

29. How will you prepare the following haloalkanes?
a) CH_3CH_2Br from alcohol
b) CH_3CH_2Cl from alkene
c) C_6H_5Cl from benzene
Write chemical equations.

30. Give reasons:
a) Alkyl halides are more reactive towards nucleophilic substitution than aryl halides.
b) Aryl halides do not undergo SN1 or SN2 easily.
c) Tertiary alkyl halides undergo SN1 reaction more easily than primary.

Music	1.Music : Learn and write in fair Note book Topics :Alankar , Kan Meend khatka Murki ,Gamak 2.Make a chart on any Musical instrument
Physical Education	खेलो में योजना' 1.योजना का अर्थ एवं उदेश्य 2.विभिन प्रकार की समितिया व उनकी जिम्मेदारिया 3.टूर्नामेंट्स-नॉक आउट,लीग या राउंड रोबिन एवं कॉम्बिनेशन 4.फिक्स्चर तैयार करने की विधि- नॉक आउट(बाई व सीडिंग तथा लीग) 5.साइक्लिक तथा -स्टेयरकेस 6.इंट्रामयुरल्स व एक्सट्राम्युरल्स (आंतरिक व बाह्य) अर्थ उदेशय व इसका महत्व 7.विशिष्ट खेल कार्यक्रम(खेल दिवस स्वास्थय दौड़ रन फॉर फन विशेष कारन क लिए दौड़ व एकता के लिए दौड़ साहसिक खेल तथा नेतृत्व प्रशिक्षण 1.साहसिक खेलों का अर्थ एवं उद्देश्य 2.गतिविधियों क्रियाकलापों के प्रकार 3.क्रियाकलापों के लिए आवश्यक सामग्री एवं सुरक्षा उपाय 4.प्राकृतिक संसाधनों की पहचान एवं प्रयोग 5.पर्यावरण संरक्षण 6.शारीरिक शिक्षा द्वारा नेताओं का सृजन पूरे पाठ्यक्रम का पुनरावलोकन करें
Maths	XII HOLIDAY HW MATHS (1).pdf

CHAPTER – 1: RELATIONS AND FUNCTIONS

MARKS WEIGHTAGE – 06 marks

NCERT Important Questions & Answers

1. **Show that the relation *R* in the set *R* of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.**
Ans:
We have $R = \{(a,b) : a \le b^2\}$, where $a, b \in R$
For reflexivity, we observe that $\frac{1}{2} \le \left(\frac{1}{2}\right)^2$ is not true.
So, R is not reflexive as $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$
For symmetry, we observe that $-1 \le 3^2$ but $3 > (-1)^2$
$\therefore (-1, 3) \in R$ but $(3, -1) \notin R$.
So, R is not symmetric.
For transitivity, we observe that$2 \le (-3)^2$ and $-3 \le (1)^2$ but $2 > (1)^2$
$\therefore (2, -3) \in R$ and $(-3,1) \in R$ but $(2, 1) \notin R$. So, R is not transitive.
Hence, R is neither reflexive, nor symmetric and nor transitive.

2. **Prove that the relation *R* in *R* defined by $R = \{(a, b): a \le b^3\}$ is neither reflexive nor symmetric nor transitive.**
Ans:
Given that $R = \{(a, b): a \le b^3\}$
It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \in R$ as $\frac{1}{2} < \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
So, R is not reflexive.
Now, (1, 2) (as $1 < 2^3 = 8$)
But $(2, 1) \notin$ R (as $2^3 > 1$)
So, R is not symmetric.
We have $\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 > \left(\frac{3}{2}\right)^3$ *and* $\frac{3}{2} < \left(\frac{6}{5}\right)^3$
But $\left(3, \frac{6}{5}\right) \notin R$ *as* $3 > \left(\frac{6}{5}\right)^3$
Therefore, R is not transitive.
Hence, R is neither reflexive nor symmetric nor transitive.

3. **Show that the relation R in the set *A* = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.**
Ans:
Given that A = {1, 2, 3, 4, 5} and R = {(a, b) : |a – b| is even}
It is clear that for any element $a \in A$, we have (which is even).
∴ R is reflexive.
Let $(a, b) \in$ R.
⇒ |a – b| is even
⇒(a – b) is even
⇒– (a – b) is even

$\Rightarrow$(b – a) is even
$\Rightarrow$ |b – a| is even
$\Rightarrow$(b, a) $\in$ R
$\therefore$**R is symmetric.**
Now, let (a, b) $\in$ R and (b, c) $\in$ R.
$\Rightarrow$ |a – b| is even and |b – c| is even
$\Rightarrow$ (a – b) is even and (b – c) is even
$\Rightarrow$ (a – c) = (a – b) + (b – c) is even (Since, sum of two even integers is even)
$\Rightarrow$ |a – c| is even
$\Rightarrow$ (a, c) $\in$ R
$\therefore$**R is transitive.**
Hence, R is an equivalence relation.
Now, all elements of the set {1, 2, 3} are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.
Similarly, all elements of the set {2, 4} are related to each other as all the elements of this subset are even.
Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

4. **Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by R = {(a, b): |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.**
Ans:
$A = \{x \in Z : 0 \le x \le 12\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and
R = {(a, b): |a – b| is a multiple of 4}
For any element a $\in$A, we have (a, a) $\in$ R $\Rightarrow$ |a – a| = 0 is a multiple of 4.
$\therefore$R is reflexive.
Now, let (a, b) $\in$ R $\Rightarrow$|a – b| is a multiple of 4.
$\Rightarrow$|–(a – b)| is a multiple of 4
$\Rightarrow$|b – a| is a multiple of 4.
$\Rightarrow$ (b, a) $\in$ R
$\therefore$R is symmetric.
Now, let (a, b), (b, c) $\in$ R.
$\Rightarrow$|a – b| is a multiple of 4 and |b – c| is a multiple of 4.
$\Rightarrow$(a – b) is a multiple of 4 and (b – c) is a multiple of 4.
$\Rightarrow$(a – b + b – c) is a multiple of 4
$\Rightarrow$(a – c) is a multiple of 4
$\Rightarrow$|a – c| is a multiple of 4
$\Rightarrow$ (a, c) $\in$R
$\therefore$ R is transitive.
Hence, R is an equivalence relation.
The set of elements related to 1 is {1, 5, 9} since
|1 – 1| = 0 is a multiple of 4
|5 – 1| = 4 is a multiple of 4
|9 – 1| = 8 is a multiple of 4

5. **Let $A = R - \{3\}$ and $B = R - \{1\}$. Prove that the function $f : A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$ is f one-one and onto ? Justify your answer.**
Ans:

Here, $A = R - \{3\}$, $B = R - \{1\}$ and $f : A \to B$ is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x, y \in A$ such that $f(x) = f(y)$

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3} \Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow -3x - 2y = -3y - 2x$

$\Rightarrow 3x - 2x = 3y - 2y \Rightarrow x = y$

Therefore, f is one- one. Let $y \in B = R - \{1\}$. Then, $y \neq 1$

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$\Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow x(1-y) = -3y + 2$

$\Rightarrow x = \frac{2-3y}{1-y} \in A \;\; [y \neq 1]$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

Therefore, f is onto. Hence, function f is one-one and onto.

6. Show that $f : [-1,1] \to R$, given by $f(x) = \frac{x}{x+2}, x \neq -2$, is one-one.

Ans:

Given that $f : [-1,1] \to R$, given by $f(x) = \frac{x}{x+2}, x \neq -2$,

Let f(x) = f(y)

$\Rightarrow \frac{x}{x+2} = \frac{y}{y+2} \Rightarrow xy + 2x = xy + 2y$

$\Rightarrow 2x = 2y \Rightarrow x = y$

Therefore, f is a one-one function.

7. Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function.

Ans:

It is given that $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$

Suppose, $f(x) = f(y)$, where $x, y \in R \Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$

It can be observed that if x is positive and y is negative, then we have

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since, x is positive and y is negative, then $x > y \Rightarrow x - y > 0$

But, $2xy$ is negative. Then, $2xy \neq x - y$.
Thus, the case of x being positive and y being negative can be ruled out.
Under a similar argument, x being negative and y being positive can also be ruled out. Therefore, x and y have to be either positive or negative.

When x and y are both positive, we have $f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$

When x and y are both negative, we have $f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$

Therefore, f is one-one. Now, let $y \in R$ such that $-1 < y < 1$.

If y is negative, then there exists $x = \frac{y}{1+y} \in R$ such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|} = \frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)} = \frac{y}{1+y-y} = y$$

If y is positive, then there exists $x = \frac{y}{1-y} \in R$ such that

$$f(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left|\frac{y}{1-y}\right|} = \frac{\frac{y}{1-y}}{1+\left(\frac{y}{1-y}\right)} = \frac{y}{1-y+y} = y$$

Therefore, f is onto. Hence, f is one-one and onto.

8. Show that the function $f : R \rightarrow R$ given by $f(x) = x^3$ is injective.
Ans:
Here, $f : R \rightarrow R$ is given as $f(x) = x^3$.
Suppose, $f(x) = f(y)$,where $x, y \in R \Rightarrow x^3 = y^3$ …(i)
Now, we need to show that $x = y$
Suppose, $x \neq y$, their cubes will also not be equal.
$x^3 \neq y^3$
However, this will be a contradiction to Eq. i).
Therefore, $x = y$. Hence, f is injective.

9. Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides $a – b$} is an equivalence relation.
Ans:
R is reflexive, as 2 divides $(a – a)$ for all $a \in$ **Z**.
Further, if $(a, b) \in$ R, then 2 divides $a – b$.
Therefore, 2 divides $b – a$.
Hence, $(b, a) \in$ R, which shows that R is symmetric.
Similarly, if $(a, b) \in$ R and $(b, c) \in$ R, then $a – b$ and $b – c$ are divisible by 2.
Now, $a – c = (a – b) + (b – c)$ is even.
So, $(a – c)$ is divisible by 2. This shows that R is transitive.
Thus, R is an equivalence relation in **Z**.

CHAPTER – 1: RELATIONS AND FUNCTIONS

MARKS WEIGHTAGE – 06 marks

Previous Years Board Exam (Important Questions & Answers)

1. **Let T be the set of all triangles in a plane with R as relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.**
Ans:
(*i*) Reflexive
R is reflexive if $T_1\ R\ T_1$
Since $T_1 \cong T_1$
$\therefore$ R is reflexive.
(*ii*) Symmetric
R is symmetric if $T_1\ R\ T_2 \Rightarrow T_2\ R\ T_1$
Since $T_1 \cong T_2 \Rightarrow T_2 \cong T_1$
$\therefore$ R is symmetric.
(*iii*) Transitive
R is transitive if $T_1\ R\ T_2$ and $T_2\ R\ T_3 \Rightarrow T_1\ R\ T_3$
Since $T_1 \cong T_2$ and $T_2 \cong T_3 \Rightarrow T_1 \cong T_3$
$\therefore$ R is transitive
From (*i*), (*ii*) and (*iii*), we get R is an equivalence relation.

2. **Let $f : N \to N$ be defined by** $f(n) = \begin{cases} \dfrac{n+1}{2}, if \ \ n \ \ is \ \ odd \\ \dfrac{n}{2}, if \ \ n \ \ is \ \ even \end{cases}$ $for \ \ all \ \ n \in N$ **. Find whether the function f is bijective.**
Ans:
Given that $f : N \to N$ be defined by $f(n) = \begin{cases} \dfrac{n+1}{2}, if \ \ n \ \ is \ \ odd \\ \dfrac{n}{2}, if \ \ n \ \ is \ \ even \end{cases}$ $for \ \ all \ \ n \in N$.
Let $x, y \in N$ and let they are odd then
$f(x) = f(y) \Rightarrow \dfrac{x+1}{2} = \dfrac{y+1}{2} \Rightarrow x = y$
If $x, y \in N$ are both even then also
$f(x) = f(y) \Rightarrow \dfrac{x}{2} = \dfrac{y}{2} \Rightarrow x = y$
If $x, y \in N$ are such that x is even and y is odd then
$f(x) = \dfrac{x+1}{2} \ \ and \ \ f(y) = \dfrac{y}{2}$
Thus, $x \neq y$ for $f(x) = f(y)$
Let $x = 6$ and $y = 5$
We get $f(6) = \dfrac{6}{2} = 3, f(5) = \dfrac{5+1}{2} = 3$
$\therefore$ $f(x) = f(y)$ but $x \neq y$ ...(*i*)
So, $f(x)$ is not one-one.
Hence, $f(x)$ is not bijective.

3. What is the range of the function $f(x) = \dfrac{|x-1|}{(x-1)}$?

Ans:

We have given $f(x) = \dfrac{|x-1|}{(x-1)}$

$$|x-1| = \begin{cases} (x-1), & if \quad x-1>0 \quad or \quad x>1 \\ -(x-1), & if \quad x-1<0 \quad or \quad x<1 \end{cases}$$

(*i*) For $x > 1$, $f(x) = \dfrac{(x-1)}{(x-1)} = 1$

(*ii*) For $x < 1$, $f(x) = \dfrac{-(x-1)}{(x-1)} = -1$

$\therefore$ Range of $f(x) = \dfrac{|x-1|}{(x-1)}$ is $\{-1, 1\}$.

4. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b)\ ;\ a, b \in Z$, and $(a - b)$ is divisible by 5.$\}$ Prove that R is an equivalence relation.

Ans:

We have provided $R = \{(a, b) : a, b \in Z$, and$(a - b)$ is divisible by 5$\}$

(*i*) As $(a - a) = 0$ is divisible by 5.

$\therefore (a, a) \in R \ \forall \ a \in R$

Hence, R is reflexive.

(*ii*) Let $(a, b) \in R$

$\Rightarrow (a - b)$ is divisible by 5.

$\Rightarrow -(b - a)$ is divisible by 5.

$\Rightarrow (b - a)$ is divisible by 5.

$\therefore (b, a) \in R$

Hence, R is symmetric.

(*iii*) Let $(a, b) \in R$ and $(b, c) \in Z$

Then, $(a - b)$ is divisible by 5 and $(b - c)$ is divisible by 5.

$(a - b) + (b - c)$ is divisible by 5.

$(a - c)$ is divisible by 5.

$\therefore (a, c) \in R$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

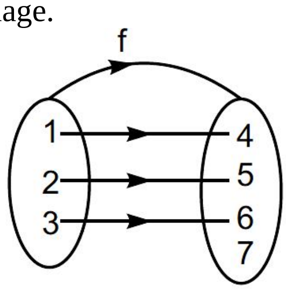
5. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let *f* = {(1, 4), (2,5), (3, 6)} be a function from A to B. State whether *f* is one-one or not.

Ans:

f is one-one because

$f(1) = 4$; $f(2) = 5$; $f(3) = 6$

No two elements of *A* have same *f* image.



6. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Show that f is one-one and onto and hence find f^{-1}.

Ans:

Let $x_1, x_2 \in A$.

Now, $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

Hence f is one-one function.

For Onto

Let $y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2 \Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$ - - - - - (i)

From above it is obvious that $\forall$ y except 1, *i.e.*, $\forall y \in B = R - \{1\}$ $\exists x \in A$

Hence f is onto function.

Thus f is one-one onto function.

It f^{-1} is inverse function of f then $f^{-1}(y) = \frac{3y-2}{y-1}$ [from (i)]

7. Show that f : N $\to$ N, given by $f(x) = \begin{cases} x+1, if \quad x \;\; is \;\; odd \\ x-1, if \quad x \;\; is \;\; even \end{cases}$ is both one-one and onto.

Ans:

For one-one

Case I : When x_1, x_2 are odd natural number.

$\therefore$ $f(x_1) = f(x_2) \Rightarrow$ $x_1+1 = x_2 +1$ $\forall$ x_1 , $x_2 \in N$

$\Rightarrow x_1 = x_2$

i.e., f is one-one.

Case II : When x_1, x_2 are even natural number

$\therefore f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

i.e., f is one-one.

Case III : When x_1 is odd and x_2 is even natural number

$f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2 - 1$

$\Rightarrow x_2 - x_1 = 2$ which is never possible as the difference of odd and even number is always odd number.

Hence in this case $f(x_1) \neq f(x_2)$

i.e., f is one-one.

Case IV: When x_1 is even and x_2 is odd natural number

Similar as case III, We can prove f is one-one

For onto:

$\therefore f(x) = x +1$ if x is odd

$= x - 1$ if x is even

$\Rightarrow$ For every even number 'y' of codomain $\exists$ odd number y - 1 in domain and for every odd number y of codomain $\exists$ even number y +1 in Domain.
i.e. f is onto function.
Hence f is one-one onto function.

8. Prove that the relation *R* in the set *A* = {5, 6, 7, 8, 9} given by *R* = {(*a*, *b*) : |*a* – *b*| , is divisible by 2}, is an equivalence relation. Find all elements related to the element 6.

Ans:

Here R is a relation defined as $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$

Reflexivity

Here $(a, a) \in R$ as $|a - a| = 0 = 0$ divisible by 2 *i.e.*, R is reflexive.

Symmetry

Let $(a, b) \in R$
$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2
$\Rightarrow a - b = \pm 2m \Rightarrow b - a = \mp 2m$
$\Rightarrow |b - a|$ is divisible by 2 $\Rightarrow (b, a) \in R$
Hence R is symmetric

Transitivity Let $(a, b), (b, c) \in R$
Now, $(a, b), (b, c) \in R \Rightarrow |a - b|, |b - c|$ are divisible by 2
$\Rightarrow a - b = \pm 2m$ and $b - c = \pm 2n$
$\Rightarrow a - b + b - c = \pm 2(m + n)$
$\Rightarrow (a - c) = \pm 2k \quad [\because k = m + n]$
$\Rightarrow (a - c) = 2k$
$\Rightarrow (a - c)$ is divisible by 2 $\Rightarrow (a, c) \in R$.
Hence R is transitive.
Therefore, R is an equivalence relation.
The elements related to 6 are 6, 8.

<u>OBJECTIVE TYPE QUESTIONS (1 MARK)</u>

1. Let R be a relation on the set L of lines defined by l_1R l_2 if l_1 is perpendicular to l_2, then relation R is
(a) reflexive and symmetric (b) symmetric and transitive
(c) equivalence relation (d) symmetric

2. Given triangles with sides T_1 : 3, 4, 5; T_2 : 5, 12, 13; T_3 : 6, 8, 10; T_4 : 4, 7, 9 and a relation R in set of triangles defined as R = {(Δ_1, Δ_2) : Δ_1 is similar to Δ_2}. Which triangles belong to the same equivalence class?
(a) T_1 and T_2 (b) T_2 and T_3 (c) T_1 and T_3 (d) T_1 and T_4.

3. Given set A = {1, 2, 3} and a relation R = {(1, 2), (2, 1)}, the relation R will be
(a) reflexive if (1, 1) is added (b) symmetric if (2, 3) is added
(c) transitive if (1, 1) is added (d) symmetric if (3, 2) is added

4. Given set A = {a, b, c}. An identity relation in set A is
(a) R = {(a, b), (a, c)} (b) R = {(a, a), (b, b), (c, c)}
(c) R = {(a, a), (b, b), (c, c), (a, c)} (d) R = {(c, a), (b, a), (a, a)}

5. A relation S in the set of real numbers is defined as xSy $\Rightarrow$ $x - y + \sqrt{3}$ is an irrational number, then relation S is
(a) reflexive (b) reflexive and symmetric (c) transitive (d) symmetric and transitive

6. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is
(a) Reflexive and symmetric (b) Transitive and symmetric
(c) Equivalence (d) Reflexive, transitive but not symmetric

7. Let L denote the set of all straight lines in a plane. Let a relation R be defined by *l* R *m* if and only if *l* is perpendicular to m for all *l, m* $\in$ L. Then R is
(a) reflexive (b) symmetric **(c)** transitive (d) none of these

8. Let N be the set of natural numbers and the function f : N $\rightarrow$ N be defined by f (n) = 2n + 3 $\forall n \in N$. Then f is
(a) surjective (b) injective **(c)** bijective (d) none of these

9. Set A has 3 elements and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B is
(a) 144 (b) 12 **(c)** 24 (d) 64

10. Let f : R $\rightarrow$ R be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and – 3, respectively, are
(a) φ, {4, – 4} (b) {3, – 3}, φ **(c)** {4, –4}, φ (d) {4, – 4, {2, – 2}

11. For real numbers x and y, define xRy if and only if x – y + 2 is an irrational number. Then the relation R is
(a) reflexive (b) symmetric **(c)** transitive (d) none of these

12. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b $\forall$ a, b $\in$ T. Then R is
(a) reflexive but not transitive (b) transitive but not symmetric
(c) equivalence (d) none of these

13. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is
(a) symmetric but not transitive (b) transitive but not symmetric
(c) neither symmetric nor transitive (d) both symmetric and transitive

14. The maximum number of equivalence relations on the set A = {1, 2, 3} are
(a) 1 (b) 2 **(c)** 3 (d) 5

15. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is
(a) reflexive (b) transitive **(c)** symmetric (d) none of these

16. Let us define a relation R in R as aRb if a ≥ b. Then R is
(a) an equivalence relation (b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive (d) neither transitive nor reflexive but symmetric.

17. Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is
(a) reflexive but not symmetric (b) reflexive but not transitive
(c) symmetric and transitive (d) neither symmetric, nor transitive

18. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is
(a) 720 (b) 120 **(c)** 0 (d) none of these

19. Let A = {1, 2, 3, ...n} and B = {a, b}. Then the number of surjections from A into B is
(a) ${}^nP_2$ (b) $2^n - 2$ **(c)** $2^n - 1$ (d) None of these

20. Let f : R → R be defined by f (x) = $\frac{1}{x}$. Then f is

(a) one-one (b) onto **(c)** bijective (d) f is not defined

21. Which of the following functions from Z into Z are bijections?

(a) f (x) = x^3 (b) f (x) = x + 2 **(c)** f (x) = 2x + 1 (d) f (x) = x2 + 1

22. Let f : [2, ∞) → R be the function defined by f (x) = $x^2 - 4x + 5$, then the range of f is

(a) R (b) [1, ∞) **(c)** [4, ∞) (d) [5, ∞)

23. Let f : N → R be the function defined by f (x) = $\frac{2x-1}{2}$ and g : Q → R be another function defined by g(x) = x + 2. Then (g o f)$\frac{3}{2}$ is

(a) 1 (b) 1 (c) $\frac{7}{2}$ (d) none of these

24. Let f : R → R be defined by $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \le 3 \\ 3x : x \le 1 \end{cases}$. Then f (– 1) + f (2) + f (4) is

(a) 9 (b) 14 (c) 5 (d) none of these

25. Let the function 'f ' : N → N be defined by f(x) = 2x + 3, x ∈ N. Then 'f ' is

(a) not onto (b) bijective function (c) many-one, into function (d) none of these

26. A relation defined in a non-empty set A, having n elements, has

(a) n relations (b) 2 relations (c) n^2 relations (d) $2n^2$ relations

27. If f (x) = x^3 and g(x) = cos 3x , then fog is

(a) x^3. cos 3x (b) cos $3x^3$ (c) $\cos^3 3x$ (d) 3cos x^3.

28. A relation R in human beings defined as R = {(a, b) : a, b human beings ; a loves b} is

(a) reflexive (b) symmetric and transitive (c) equivalence (d) neither of these

29. Consider the set A = {1, 2, 3} and R be the smallest equivalence relation on A, then R = ________

30. The domain of the function f : R→R defined by f (x) = $\sqrt{x^2 - 3x + 2}$ is ________.

31. The domain of the function f : R→R defined by f (x) = $\sqrt{4 - x^2}$ is ________.

32. Consider the set A containing n elements. Then, the total number of injective functions from A onto itself is ________.

33. Let Z be the set of integers and R be the relation defined in Z such that aRb if a – b is divisible by 3. Then R partitions the set Z into ________ pairwise disjoint subsets.

34. Consider the set A = {1, 2, 3} and the relation R = {(1, 2), (1, 3)}. R is a __________relation.

35. Let the relation R be defined in N by aRb if 2a + 3b = 30. Then R = ______.

36. Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b) : $|a^2 - b^2| < 8$. Then R is given by _______.

37. Let R be a relation defined as R = {(x, x), (y, y), (z, z), (x, z)} in set A = {x, y, z} then R is ______________ (reflexive/symmetric) relation.

38. Let A = {1, 2, 3, 4} and B = {a, b, c}. Then number of one-one functions from A to B are ________.

39. If n(a) = p, then number of bijective functions from set A to A are _______.

40. If $f(x) = \frac{x-1}{|x-1|}$, $x(\neq 1) \in$ R then range of 'f ' is _____.

INVERSE TRIGONOMETRIC FUNCTIONS

CHAPTER – 2: INVERSE TRIGONOMETRIC FUNCTIONS

MARKS WEIGHTAGE – 02 marks

QUICK REVISION (Important Concepts & Formulae)

Inverse Trigonometrical Functions

A function $f : A \to B$ is invertible if it is a bijection. The inverse of f is denoted by f^{-1} and is defined as $f^{-1}(y) = x \Leftrightarrow f(x) = y$.

☞ Clearly, domain of f^{-1} = range of f and range of f^{-1} = domain of f.

☞ The inverse of sine function is defined as $\sin^{-1}x = \theta \Leftrightarrow \sin q = x$, where $\theta \in [-\pi/2, \pi/2]$ and $x \in [-1, 1]$.

☞ Thus, sin –1 x has infinitely many values for given $x \in [-1, 1]$

☞ There is one value among these values which lies in the interval $[-\pi/2, \pi/2]$. This value is called the principal value.

Domain and Range of Inverse Trigonometrical Functions

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, 0) \cup (0, \pi/2]$

Properties of Inverse Trigonometrical Functions

☞ $\sin^{-1}(\sin\theta) = \theta$ and $\sin(\sin^{-1}x) = x$, provided that $-1 \le x \le 1$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

☞ $\cos^{-1}(\cos\theta) = \theta$ and $\cos(\cos^{-1}x) = x$, provided that $-1 \le x \le 1$ and $0 \le \theta \le \pi$

☞ $\tan^{-1}(\tan\theta) = \theta$ and $\tan(\tan^{-1}x) = x$, provided that $-\infty \le x \le \infty$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

☞ $\cot^{-1}(\cot\theta) = \theta$ and $\cot(\cot^{-1}x) = x$, provided that $-\infty < x < \infty$ and $0 < \theta < \pi$.

☞ $\sec^{-1}(\sec\theta) = \theta$ and $\sec(\sec^{-1}x) = x$

☞ $\text{cosec}^{-1}(\text{cosec}\theta) = \theta$ and $\text{cosec}(\text{cosec}^{-1}x) = x$,

☞ $\sin^{-1}x = \cos ec^{-1}\frac{1}{x}$ *or* $\cos ec^{-1}x = \sin^{-1}\frac{1}{x}$

- ☞ $\cos^{-1} x = \sec^{-1}\frac{1}{x}$ *or* $\sec^{-1} x = \cos^{-1}\frac{1}{x}$
- ☞ $\tan^{-1} x = \cot^{-1}\frac{1}{x}$ *or* $\cot^{-1} x = \tan^{-1}\frac{1}{x}$
- ☞ $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1}\frac{1}{x}$
- ☞ $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x}$
- ☞ $\tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x} = \sec^{-1}\sqrt{1+x^2} = \operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}$
- ☞ $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, where -1 \le x \le 1$
- ☞ $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, where -\infty \le x \le \infty$
- ☞ $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, where \;\; x \le -1 \;\; or \;\; x \ge 1$
- ☞ $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), if \;\; xy < 1$
- ☞ $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), if \;\; xy > 1$
- ☞ $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$
- ☞ $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), if \;\; x, y \ge 0, x^2 + y^2 \le 1$
- ☞ $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), if \;\; x, y \ge 0, x^2 + y^2 \le 1$
- ☞ $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), if \;\; x, y \ge 0, x^2 + y^2 > 1$
- ☞ $\sin^{-1} x - \sin^{-1} y = \pi - \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), if \;\; x, y \ge 0, x^2 + y^2 > 1$
- ☞ $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right), if \;\; x, y \ge 0, x^2 + y^2 \le 1$

☞ $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right), if \quad x,y\geq 0, x^2+y^2\leq 1$

☞ $\cos^{-1}x+\cos^{-1}y=\pi-\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right), if \quad x,y\geq 0, x^2+y^2>1$

☞ $\cos^{-1}x-\cos^{-1}y=\pi-\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right), if \quad x,y\geq 0, x^2+y^2>1$

☞ $\sin^{-1}(-x)=-\sin^{-1}x, \quad \cos^{-1}(-x)=\pi-\cos^{-1}x$

☞ $\tan^{-1}(-x)=-\tan^{-1}x, \quad \cot^{-1}(-x)=\pi-\cot^{-1}x$

☞ $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right), \qquad 2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$

☞ $2\tan^{-1}x=\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)=\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)=\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

☞ $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right), \quad 3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right)$

☞ $3\tan^{-1}x=\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$

CHAPTER – 2: INVERSE TRIGONOMETRIC FUNCTIONS

MARKS WEIGHTAGE – 02 marks

NCERT Important Questions & Answers

1. Find the values of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Ans:

Let $\tan^{-1}(1)=x \Rightarrow \tan x = 1 = \tan\frac{\pi}{4} \Rightarrow x=\frac{\pi}{4}$ *where* $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y \Rightarrow \cos y=-\frac{1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\frac{2\pi}{3}$ $\quad(\because \cos(\pi-\theta)=-\cos\theta)$

$\Rightarrow y=\frac{2\pi}{3}$ *where* $y\in[0,\pi]$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z \Rightarrow \sin z=-\frac{1}{2}=-\sin\frac{\pi}{6}=\sin\left(-\frac{\pi}{6}\right)\Rightarrow z=-\frac{\pi}{6}$ *where* $z\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)=x+y+z=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$

$$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$$

2. Prove that $3\sin^{-1}x=\sin^{-1}(3x-4x^3), x\in\left(-\frac{1}{2},\frac{1}{2}\right)$

Ans:

Let $\sin^{-1}x=\theta \Rightarrow x=\sin\theta$, *then*

We know that $\sin 3\theta = 3\sin\theta-4\sin^3\theta$

$\therefore 3\theta=\sin^{-1}(3\sin\theta-4\sin^3\theta)=\sin^{-1}(3x-4x^3)$

$\Rightarrow 3\sin^{-1}x=\sin^{-1}(3x-4x^3)$

3. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

Ans:

Given $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

$$LHS=\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}.\frac{7}{24}}\right) \qquad \left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$=\tan^{-1}\left(\frac{\frac{48+77}{264}}{1-\frac{14}{264}}\right)=\tan^{-1}\left(\frac{\frac{125}{264}}{\frac{264-14}{264}}\right)=\tan^{-1}\left(\frac{\frac{125}{264}}{\frac{250}{264}}\right)=\tan^{-1}\frac{125}{250}=\tan^{-1}\frac{1}{2}=RHS$$

4. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Ans:

$$Given\ \ 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$$

$$LHS=2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right)+\tan^{-1}\frac{1}{7}\qquad\left(\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$$

$$=\tan^{-1}\frac{1}{1-\frac{1}{4}}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}.\frac{1}{7}}\right)\qquad\left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$=\tan^{-1}\left(\frac{\frac{28+3}{21}}{1-\frac{4}{21}}\right)=\tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)=\tan^{-1}\frac{31}{17}=RHS$$

5. Simplify : $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x\neq 0$

Ans:

Let $x=\tan\theta$, then $\theta=\tan^{-1}x$ ………………… (i)

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{2}=\tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}=\tan^{-1}\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}$$

$$=\tan^{-1}\frac{\sec\theta-1}{\tan\theta}=\tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)=\tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\qquad\left[\begin{array}{l}\because 1-\cos\theta=2\sin^2\frac{\theta}{2}\\ and\ \ \sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\end{array}\right]$$

$$=\tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)=\tan^{-1}\tan\frac{\theta}{2}=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x\quad\text{[using (i)]}$$

6. Simplify : $\tan^{-1}\frac{1}{\sqrt{x^2-1}},|x|>1$

Ans:

Let $x=\sec\theta$, then $\theta=\sec^{-1}x$ ………………… (i)

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}}=\tan^{-1}\frac{1}{\sqrt{\sec^2\theta-1}}=\tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$$

$$=\tan^{-1}\frac{1}{\tan\theta}=\tan^{-1}(\cot\theta)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)\left(\because \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta\right)$$

$$= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \sec^{-1} x \qquad \text{[using (i)]}$$

7. Simplify : $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), 0 < x < \pi$

Ans:

$$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right)$$

(inside the bracket divide numerator and denominator by cos x)

$$= \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right) \qquad \left(\because \tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}\right)$$

$$= \frac{\pi}{4} - x$$

8. Simplify : $\tan \frac{1}{2}\left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0 \ \ and \ \ xy < 1$

Ans:

$$\tan \frac{1}{2}\left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0 \ \ and \ \ xy < 1$$

$$\left[\because 2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} \ \ and \ \ 2\tan^{-1} y = \cos^{-1} \frac{1-y^2}{1+y^2}\right]$$

$$= \tan \frac{1}{2}\left[(2\tan^{-1} x + 2\tan^{-1} y)\right] = \tan\left[\frac{1}{2}.2(\tan^{-1} x + \tan^{-1} y)\right] = \tan(\tan^{-1} x + \tan^{-1} y)$$

$$= \tan\left(\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right) \qquad \left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$= \frac{x+y}{1-x.y}$$

9. Find the value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$**.**

Ans:

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right) where, \frac{5\pi}{6} \in [0, \pi]$$

$$\therefore \cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6} \ \ (\because \cos(2\pi - \theta) = \cos\theta)$$

10. Prove that $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$

Ans:

Given $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$

Let $\cos^{-1} \frac{12}{13} = x \Rightarrow \cos x = \frac{12}{13}$

$$\therefore \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$\Rightarrow x=\sin^{-1}\frac{5}{13}$

$LHS=\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{3}{5}$

$=\sin^{-1}\left(\frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}+\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}\right)$ $\left[\because \sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\right]$

$=\sin^{-1}\left(\frac{5}{13}\sqrt{\frac{16}{25}}+\frac{3}{5}\sqrt{\frac{144}{169}}\right)=\sin^{-1}\left(\frac{5}{13}\times\frac{4}{5}+\frac{3}{5}\times\frac{12}{13}\right)$

$=\sin^{-1}\left(\frac{20}{65}+\frac{36}{65}\right)=\sin^{-1}\frac{56}{65}=RHS$

11. Prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Ans:

$RHS=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

$Let\ \sin^{-1}\frac{5}{13}=x\Rightarrow \sin x=\frac{5}{13}$

$\therefore \cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

$\Rightarrow \tan x=\frac{\sin x}{\cos x}=\frac{\frac{5}{13}}{\frac{12}{13}}=\frac{5}{12}\Rightarrow x=\tan^{-1}\frac{5}{12}$

$Let\ \cos^{-1}\frac{3}{5}=y\Rightarrow \cos y=\frac{3}{5}$

$\therefore \sin y=\sqrt{1-\cos^2 y}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\Rightarrow \tan y=\frac{\sin x}{\cos x}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}\Rightarrow y=\tan^{-1}\frac{4}{3}$

then the equation becomes $\tan^{-1}\frac{63}{16}=x+y$

$\Rightarrow \tan^{-1}\frac{63}{16}=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$

$RHS=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}.\frac{4}{3}}\right)$ $\left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$

$=\tan^{-1}\left(\frac{\frac{15+48}{36}}{1-\frac{20}{36}}\right)=\tan^{-1}\left(\frac{\frac{63}{36}}{\frac{16}{36}}\right)=\tan^{-1}\left(\frac{63}{16}\right)=LHS$

12. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

Ans:

$$LHS = \left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}\right)+\left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}.\frac{1}{8}}\right) \qquad \left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$= \tan^{-1}\left(\frac{\frac{7+5}{35}}{1-\frac{1}{35}}\right)+\tan^{-1}\left(\frac{\frac{8+3}{24}}{1-\frac{1}{24}}\right)=\tan^{-1}\left(\frac{\frac{12}{35}}{\frac{30}{35}}\right)+\tan^{-1}\left(\frac{\frac{11}{24}}{\frac{23}{24}}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right)+\tan^{-1}\left(\frac{11}{23}\right)=\tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{\frac{138+187}{391}}{1-\frac{66}{391}}\right)=\tan^{-1}\left(\frac{\frac{325}{391}}{\frac{325}{391}}\right)=\tan^{-1}(1)=\frac{\pi}{4}=RHS$$

13. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

Ans:

Given $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

$$LHS = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$= \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\times\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right) \text{ (by rationalizing the denominator)}$$

$$= \cot^{-1}\left(\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}\right)=\cot^{-1}\left(\frac{1+\sin x+1-\sin x+2\sqrt{1-\sin^2 x}}{1+\sin x-1+\sin x}\right)$$

$$= \cot^{-1}\left(\frac{2+2\cos x}{\sin x}\right)=\cot^{-1}\left(\frac{2(1+\cos x)}{2\sin x}\right)=\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$= \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \qquad \left(\because 1+\cos x = 2\cos^2\frac{x}{2} \text{ and } \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right)$$

$$= \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right) == \cot^{-1}\left(\cot\frac{x}{2}\right)=\frac{x}{2}=RHS$$

14. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

Ans:

Let $x = \cos y \Rightarrow y=\cos^{-1}x$

$$LHS=\tan^{-1}\left(\frac{\sqrt{1+\cos y}-\sqrt{1-\cos y}}{\sqrt{1+\cos y}+\sqrt{1-\cos y}}\right)=\tan^{-1}\left(\frac{2\cos\frac{y}{2}-2\sin\frac{y}{2}}{2\cos\frac{y}{2}+2\sin\frac{y}{2}}\right)$$

$$\left(\because 1+\cos y=2\cos^2\frac{y}{2}\ \ and\ \ 1-\cos y=2\sin^2\frac{y}{2}\right)$$

$$=\tan^{-1}\left(\frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}\right)=\tan^{-1}\left(\frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}\right)=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{y}{2}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

$$\left(\because \tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}\right)$$

15. Solve for x: $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$

Ans:

Given $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$

$$\Rightarrow 2\tan^{-1}\frac{1-x}{1+x}=\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right)=\tan^{-1}x \qquad \left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{2\left(\frac{1-x}{1+x}\right)}{\frac{(1+x)^2-(1-x)^2}{(1+x)^2}}\right)=\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2}\right)=\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{2(1-x^2)}{1+2x+x^2-1+2x-x^2}\right)=\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{2(1-x^2)}{4x}\right)=\tan^{-1}x\ \Rightarrow \tan^{-1}\left(\frac{1-x^2}{2x}\right)=\tan^{-1}x$$

$$\Rightarrow \frac{1-x^2}{2x}=x\Rightarrow 1-x^2=2x^2\Rightarrow 1=3x^2\Rightarrow x^2=\frac{1}{3}\Rightarrow x=\pm\frac{1}{\sqrt{3}}$$

$$\left[\because x>0\ \text{ given, so we do not take } x=-\frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow x=\frac{1}{\sqrt{3}}$$

16. Solve for x: $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

Ans:

Given $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\operatorname{cosec}x) \qquad \left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right)=\tan^{-1}\left(\frac{2}{\sin x}\right)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x}=\frac{2}{\sin x}\Rightarrow \frac{\cos x}{\sin x}=1$$

$$\Rightarrow \cot x=1\Rightarrow \cot x=\cot\frac{\pi}{4}\Rightarrow x=\frac{\pi}{4}$$

17. Solve for x: $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Ans:

Given $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

$$\Rightarrow -2\sin^{-1}x=\frac{\pi}{2}-\sin^{-1}(1-x)\Rightarrow -2\sin^{-1}x=\cos^{-1}(1-x)$$

$$\left[\because \sin^{-1}(1-x)+\cos^{-1}(1-x)=\frac{\pi}{2}\right]$$

$$\Rightarrow \cos\left(-2\sin^{-1}x\right)=1-x$$

$$\Rightarrow \cos\left(2\sin^{-1}x\right)=1-x \qquad [\because \cos(-x)=\cos x]$$

$$\Rightarrow 1-2\sin^2\left(\sin^{-1}x\right)=1-x \qquad [\because \cos 2x=1-2\sin^2 x]$$

$$\Rightarrow 1-2\left[\sin\left(\sin^{-1}x\right)\right]^2=1-x$$

$$\Rightarrow 1-2x^2=1-x\Rightarrow 2x^2-x=0$$

$$\Rightarrow x(2x-1)=0\Rightarrow x=0 \;\; or \;\; 2x-1=0$$

$$\Rightarrow x=0 \;\; or \;\; x=\frac{1}{2}$$

But $x=\frac{1}{2}$ does not satisfy the given equation, so x = 0.

18. Simplify: $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

Ans:

Given $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{\frac{x}{y}-1}{\frac{x}{y}+1}\right)$

$$=\tan^{-1}\left(\frac{x}{y}\right)-\left(\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}1\right) \qquad \left(\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+x.y}\right)\right)$$

$$\Rightarrow \tan^{-1}1=\frac{\pi}{4}$$

19. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right),-\frac{\pi}{2}<x<\frac{\pi}{2}$ **in the simplest form.**

Ans:

Given $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right),-\frac{\pi}{2}<x<\frac{\pi}{2}$

$$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right)$$

$$\left(\because 1-\cos x = \cos^2\frac{x}{2}-\sin^2\frac{x}{2}, \sin^2\frac{x}{2}+\cos^2\frac{x}{2}=1 \ \ \textit{and} \ \ \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right) = \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{\pi}{4}+\frac{x}{2}$$

20. Simplify : $\cot^{-1}\frac{1}{\sqrt{x^2-1}}, |x|>1$

Ans:

Let $x = \sec\theta$, then $\theta = \sec^{-1}x$ ………………… (i)

$$\cot^{-1}\frac{1}{\sqrt{x^2-1}} = \cot^{-1}\frac{1}{\sqrt{\sec^2\theta-1}} = \cot^{-1}\frac{1}{\sqrt{\tan^2\theta}}$$

$$= \cot^{-1}\frac{1}{\tan\theta} = \cot^{-1}(\cot\theta) = \theta = \sec^{-1}x$$

21. Prove that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

Ans:

Let $\sin^{-1}\frac{3}{5} = x$ and $\sin^{-1}\frac{8}{17} = y$

Therefore $\sin x = \frac{3}{5}$ and $\sin y = \frac{8}{17}$

Now, $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

and $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\left(\frac{8}{17}\right)^2} = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$

We have $\cos(x-y) = \cos x\cos y + \sin x\sin y = \frac{4}{5}\times\frac{15}{17}+\frac{3}{5}\times\frac{8}{17} = \frac{60}{85}+\frac{24}{85} = \frac{84}{85}$

$\Rightarrow x-y = \cos^{-1}\frac{84}{85}$

$\Rightarrow \sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

22. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16} = \pi$

Ans:

Let $\sin^{-1}\frac{12}{13} = x$, $\cos^{-1}\frac{4}{5} = y$ and $\tan^{-1}\frac{63}{16} = z$

Then $\sin x = \frac{12}{13}$, $\cos y = \frac{4}{5}$ and $\tan z = \frac{63}{16}$

Now, $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$

and $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$$\Rightarrow \tan x = \frac{\sin x}{\cos x} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5} \quad and \quad \tan y = \frac{\sin y}{\cos y} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x.\tan y} = \frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}.\frac{3}{4}} = \frac{\frac{48+15}{20}}{1-\frac{36}{20}} = \frac{\frac{63}{20}}{-\frac{16}{20}} = -\frac{63}{16} = -\tan z$$

$\Rightarrow \tan(x+y) = -\tan z = \tan(-z) = \tan(\pi - z)$

$\Rightarrow x+y = \pi - z$

$\Rightarrow x+y+z = \pi$

$\Rightarrow \sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

23. Simplify: $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), if\ \frac{a}{b}\tan x > -1$

Ans:

$$\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = \tan^{-1}\left(\frac{\frac{a\cos x - b\sin x}{b\cos x}}{\frac{b\cos x + a\sin x}{b\cos x}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1+\frac{a}{b}\tan x}\right) = \tan^{-1}\frac{a}{b} - \tan^{-1}(\tan x) = \tan^{-1}\frac{a}{b} - x$$

CHAPTER – 2: INVERSE TRIGONOMETRIC FUNCTIONS

MARKS WEIGHTAGE – 02 marks

Previous Years Board Exam (Important Questions & Answers)

1. Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

Ans:

$$\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]$$

$$=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]=\sin\frac{\pi}{2}=1$$

2. Write the value of cot ($\tan^{-1}a+\cot^{-1}a$).

Ans:

$$\cot\left(\tan^{-1}a+\cot^{-1}a\right)=\cot\left(\frac{\pi}{2}-\cot^{-1}a+\cot^{-1}a\right)\ =\cot\frac{\pi}{2}=0$$

3. Find the principal values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$**.**

Ans:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left(\cos\left(\pi+\frac{\pi}{6}\right)\right)$$

$$=\cos^{-1}\left(-\cos\frac{\pi}{6}\right)=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

4. Find the principal values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Ans:

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)=\tan^{-1}\left(\tan\left(\pi-\frac{\pi}{4}\right)\right)$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)=\tan^{-1}(-1)=-\frac{\pi}{4}$$

5. Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

Ans:

$$LHS=\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$$

$$=\frac{\tan\frac{\pi}{4}+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan\frac{\pi}{4}\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}+\frac{\tan\frac{\pi}{4}-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1+\tan\frac{\pi}{4}\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$$

$$=\frac{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}+\frac{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$$

$$=\frac{\left[1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2+\left[1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2}{\left[1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]}$$

$$=\frac{2+2\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}=\frac{2\left(1+\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right)}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$$

$$=\frac{2}{\cos 2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}=\frac{2}{\cos\left(\cos^{-1}\frac{a}{b}\right)}=\frac{2}{\frac{a}{b}}=\frac{2b}{a}=RHS$$

6. Prove that $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$

Ans:

$$\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}=\frac{\pi}{2}-\sin^{-1}\frac{16}{65}=\cos^{-1}\frac{16}{65}$$

Let $\sin^{-1}\frac{4}{5}=x$ and $\sin^{-1}\frac{5}{13}=y$

Therefore $\sin x=\frac{4}{5}$ and $\sin y=\frac{5}{13}$

Now, $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

and $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{1-\frac{25}{144}}=\frac{12}{13}$

We have $\cos(x+y)=\cos x\cos y-\sin x\sin y=\frac{3}{5}\times\frac{12}{13}-\frac{4}{5}\times\frac{5}{13}=\frac{36}{65}-\frac{20}{65}=\frac{16}{65}$

$$\Rightarrow x+y=\cos^{-1}\frac{16}{65}$$

$$\Rightarrow \sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}=\cos^{-1}\frac{16}{65}$$

7. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}$

Ans:

$$LHS=\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}$$

$$=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}.\frac{2}{9}}\right)=\tan^{-1}\left(\frac{\frac{9+8}{36}}{1-\frac{2}{36}}\right)=\tan^{-1}\left(\frac{\frac{17}{36}}{\frac{34}{36}}\right)\qquad\left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$=\tan^{-1}\frac{17}{34}=\tan^{-1}\frac{1}{2}=\frac{1}{2}\left(2\tan^{-1}\frac{1}{2}\right)$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}\right)=\frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right) \qquad \left[\because 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{\frac{3}{4}}{\frac{5}{4}}\right)=\frac{1}{2}\cos^{-1}\frac{3}{5}=RHS$$

8. Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$

Ans:

$$LHS=\tan^{-1}\sqrt{x}=\frac{1}{2}\left(2\tan^{-1}\sqrt{x}\right)$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-\left(\sqrt{x}\right)^2}{1+\left(\sqrt{x}\right)^2}\right) \qquad \left[\because 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=RHS$$

9. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

Ans:

$$LHS=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)=\frac{9}{4}\cos^{-1}\frac{1}{3}$$

$$Let\ \ \cos^{-1}\frac{1}{3}=x\Rightarrow\cos x=\frac{1}{3}\Rightarrow\sin x=\sqrt{1-\cos^2 x}$$

$$\Rightarrow\sin x=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$$

$$\Rightarrow x=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\Rightarrow\cos^{-1}\frac{1}{3}=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\therefore\frac{9}{4}\cos^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=RHS$$

10. Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$

Ans:

$$\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$$

$$=\tan^{-1}\left(\tan\frac{\pi}{3}\right)-\sec^{-1}\left(-\sec\frac{\pi}{3}\right)$$

$$=\frac{\pi}{3}-\sec^{-1}\left(\sec\left(\pi-\frac{\pi}{3}\right)\right)=\frac{\pi}{3}-\sec^{-1}\left(\sec\frac{2\pi}{3}\right)$$

$$=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$$

11. Prove that : $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$

Ans:

Let $\sin^{-1}\frac{3}{5}=x$ and $\cot^{-1}\frac{3}{2}=y$

Then $\sin x=\frac{3}{5}$ and $\cot y=\frac{3}{2}$

Now $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

and $\sin y=\frac{1}{\sqrt{1+\cot^2 y}}=\frac{1}{\sqrt{1+\left(\frac{3}{2}\right)^2}}=\frac{1}{\sqrt{1+\frac{9}{4}}}=\frac{1}{\sqrt{\frac{13}{4}}}=\frac{2}{\sqrt{13}}$

$\Rightarrow \cos y=\frac{3}{\sqrt{13}}$

$LHS=\cos(x+y)=\cos x\cos y-\sin x\sin y=\frac{4}{5}\times\frac{3}{\sqrt{13}}-\frac{3}{5}\times\frac{2}{\sqrt{13}}$

$=\frac{12}{5\sqrt{13}}-\frac{6}{5\sqrt{13}}=\frac{6}{5\sqrt{13}}=RHS$

Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$

Ans:

$Let\ \ 2\tan^{-1}\frac{1}{5}=x\Rightarrow\tan^{-1}\frac{1}{5}=\frac{x}{2}\Rightarrow\tan\frac{x}{2}=\frac{1}{5}$

$\tan\left(2\tan^{-1}\frac{1}{5}\right)=\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}=\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}=\frac{2}{5}\times\frac{25}{24}=\frac{5}{12}$

12. Find the value of the following: $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0\ \ and\ \ xy<1$

Ans:

$Let\ \ x=\tan\alpha\ \ and\ \ y=\tan\beta\Rightarrow\alpha=\tan^{-1}x, \beta=\tan^{-1}y$

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$=\tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\alpha}{1+\tan^2\alpha}+\cos^{-1}\frac{1-\tan^2\beta}{1+\tan^2\beta}\right]$

$=\tan\frac{1}{2}\left[\sin^{-1}(\sin 2\alpha)+\cos^{-1}(\cos 2\beta)\right]\quad\left[\because\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}\ \ and\ \ \cos 2\beta=\frac{1-\tan^2\beta}{1+\tan^2\beta}\right]$

$=\tan\frac{1}{2}\left[2\alpha+2\beta\right]=\tan\left[\alpha+\beta\right]=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}=\frac{x+y}{1-xy}$

13. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

Ans:

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$=\tan^{-1}\left[2\sin\left(2\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\left(\frac{\pi}{3}\right)\right]=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$$

$$=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$$

14. Prove that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

Ans:

Let $\sin^{-1}\frac{3}{4}=\alpha\Rightarrow\sin\alpha=\frac{3}{4}$

$$\Rightarrow\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{3}{4}\qquad\left[\because\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}\right]$$

$$\Rightarrow 3+3\tan^2\frac{\alpha}{2}=8\tan\frac{\alpha}{2}\Rightarrow 3\tan^2\frac{\alpha}{2}-8\tan\frac{\alpha}{2}+3=0$$

$$\Rightarrow\tan\frac{\alpha}{2}=\frac{8\pm\sqrt{64-36}}{6}=\frac{8\pm\sqrt{28}}{6}$$

$$\Rightarrow\tan\frac{\alpha}{2}=\frac{8\pm2\sqrt{7}}{6}\Rightarrow\tan\frac{\alpha}{2}=\frac{4\pm\sqrt{7}}{3}$$

$$\Rightarrow\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$$

15. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$**, then prove that** $\sin y=\tan^2\frac{x}{2}$

Ans:

$$y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$$

$$\Rightarrow y=\frac{\pi}{2}-\tan^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)=\frac{\pi}{2}-2\tan^{-1}\left(\sqrt{\cos x}\right)$$

$$\Rightarrow y=\frac{\pi}{2}-\cos^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)\qquad\left[\because 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$\Rightarrow y=\sin^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)$$

$$\Rightarrow\sin y=\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}$$

16. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$**, then find the value of x.**

Ans:

$$\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1\ \Rightarrow\sin^{-1}\frac{1}{5}+\cos^{-1}x=\sin^{-1}1$$

$$\Rightarrow\sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}\Rightarrow\sin^{-1}\frac{1}{5}=\frac{\pi}{2}-\cos^{-1}x=\sin^{-1}x\qquad\Rightarrow x=\frac{1}{5}$$

17. Prove that $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

Ans:

$$LHS=2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}$$

$$=2\tan^{-1}\frac{1}{5}+2\tan^{-1}\frac{1}{8}+\sec^{-1}\frac{5\sqrt{2}}{7}=2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)+\sec^{-1}\frac{5\sqrt{2}}{7}$$

$$=2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}.\frac{1}{8}}\right)+\tan^{-1}\sqrt{\left(\frac{5\sqrt{2}}{7}\right)^2-1}$$

$$=2\tan^{-1}\left(\frac{\frac{8+5}{40}}{1-\frac{1}{40}}\right)+\tan^{-1}\sqrt{\frac{50}{49}-1}$$

$$=2\tan^{-1}\left(\frac{\frac{13}{40}}{\frac{39}{40}}\right)+\tan^{-1}\sqrt{\frac{1}{49}}=2\tan^{-1}\left(\frac{13}{39}\right)+\tan^{-1}\frac{1}{7}$$

$$=2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right)+\tan^{-1}\frac{1}{7}\qquad\left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$=\tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}.\frac{1}{7}}\right)\qquad\left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$=\tan^{-1}\left(\frac{\frac{21+4}{28}}{1-\frac{3}{28}}\right)=\tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)=\tan^{-1}1=\frac{\pi}{4}$$

18. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$ **then write the value of** $x+y+xy$**.**

Ans:

$$\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$$

$$\Rightarrow\tan^{-1}\left(\frac{x+y}{1-x.y}\right)=\frac{\pi}{4}\Rightarrow\frac{x+y}{1-x.y}=\tan\frac{\pi}{4}=1$$

$$\Rightarrow x+y=1-xy\Rightarrow x+y+xy=1$$

OBJECTIVE TYPE QUESTIONS (1 MARK)

1. Which of the following corresponds to the principal value branch of $\tan^{-1}x$?

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (d) $(0, \pi)$

2. Which of the following is the principal value branch of $\cos^{-1}x$?

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ (c) $(0, \pi)$ (d) $[0, \pi]$

3. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ (b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. The principal value branch of $\sec^{-1}x$ is

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ (b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (c) $(0, \pi)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

5. One branch of $\cos^{-1}$ other than the principal value branch corresponds to

(a) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (b) $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$ (c) $(0, \pi)$ (d) $[2\pi, 3\pi]$

6. The value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$

(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{10}$

7. The principal value of the expression $\cos^{-1}[\cos(-680°)]$ is

(a) $\frac{2\pi}{9}$ (b) $\frac{-2\pi}{9}$ (c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

8. The value of $\cot(\sin^{-1}x)$ is

(a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

9. If $\tan^{-1}x = \frac{\pi}{10}$ for some $x \in R$, then the value of $\cot^{-1}x$ is

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$

10. The domain of $\sin^{-1} 2x$ is

(a) $[0, 1]$ (b) $[-1, 1]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-2, 2]$

11. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $-\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$

12. If $3\tan^{-1} x + \cot^{-1} x = \pi$, then x equals

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

13. The domain of the function $\cos^{-1}(2x - 1)$ is

(a) [0, 1] (b) [–1, 1] (c) (–1, 1) (d) $[0, \pi]$

14. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1, 2] (b) [–1, 1] (c) [0, 1] (d) none of these

15. If $\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1} x\right) = 0$, then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 0 (d) 1

16. The value of $\sin(2\tan^{-1}(0.75))$ is equal to

(a) 0.75 (b) 1.5 (c) 0.96 (d) sin 1.5

17. The value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$ is

(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

18. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

19. The value of the expression $2\sec^{-1} 2 + \sin^{-1}\frac{1}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

20. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ equals

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) π

21. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where a, x $\in$]0, 1, then the value of x is

(a) 0 (b) $\frac{a}{2}$ (c) a (d) $\frac{2a}{1-a^2}$

22. The greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively

(a) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$ (b) $\frac{\pi}{2}$ and $\frac{-\pi}{2}$ (c) $\frac{\pi^2}{4}$ and $\frac{-\pi^2}{4}$ (d) $\frac{\pi^2}{4}$ and 0

23. Let $\theta = \sin^{-1}(\sin(-600°))$, then value of θ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

24. The domain of the function $y = \sin^{-1}(-x^2)$ is

(a) [0, 1] (b) (0, 1) (c) [–1, 1] (d) φ

25. The domain of $y = \cos^{-1}(x^2 - 4)$ is

(a) [3, 5] (b) [0, π] (c) $\left[-\sqrt{5}, -\sqrt{3}\right] \cap \left[-\sqrt{5}, \sqrt{3}\right]$ (d) $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

26. The domain of the function defined by $f(x) = \sin^{-1}x + \cos x$ is

(a) [–1, 1] (b) [–1, π + 1] (c) $(-\infty, \infty)$ (d) φ

27. The value of $\sin(2\sin^{-1}(0.6))$ is

(a) 0.48 (b) 0.96 (c) 1.2 (d) sin 1.2

28. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then value of $\cos^{-1}x + \cos^{-1}y$ is

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{2\pi}{3}$

29. The value of $\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right)$ is

(a) $\frac{19}{8}$ (b) $\frac{8}{19}$ (c) $\frac{19}{12}$ (d) $\frac{3}{4}$

30. The value of the expression $\sin[\cot^{-1}(\cos(\tan^{-1}1))]$ is

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

31. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\frac{1}{\sqrt{3}}$ has

(a) no solution (b) unique solution (c) infinite number of solutions (d) two solutions

32. If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$, then

(a) $\alpha = \frac{-\pi}{2}, \beta = \frac{\pi}{2}$ (b) $\alpha = 0, \beta = \pi$ (c) $\alpha = \frac{-\pi}{2}, \beta = \frac{3\pi}{2}$ (d) $\alpha = 0, \beta = 2\pi$

33. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals

(a) 0 (b) 1 (c) 6 (d) 12

34. The value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ is

(a) 5 (b) 11 (c) 13 (d) 15

35. The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is

(a) 0 (b) 1 (c) 2 (d) Infinite

36. The principal value of $\cos^{-1}x$, for $x = \frac{\sqrt{3}}{2}$ is ____________

37. The value of $\tan^{-1}\sin\frac{-\pi}{2}$ is ____________

38. The value of $\cos^{-1}\cos\frac{13\pi}{6}$ is ____________

39. The value of $\tan^{-1}\tan\frac{9\pi}{8}$ is ____________

40. The value of tan ($\tan^{-1}(-4)$) is ____________

41. The value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is ____________

42. The value of $\sin^{-1}(\cos(\sin^{-1}\frac{\sqrt{3}}{2}))$ is ____________

43. The value of $\sec\left(\tan^{-1}\frac{y}{2}\right)$ is ____________

44. The value of $\sin 2\cot^{-1}\frac{-5}{12}$ is ____________

45. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is___________.

46. The value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is___________.

47. If $\cos(\tan^{-1}x + \cot^{-1}\sqrt{3}) = 0$, then value of x is___________.

48. The set of values of $\sec^{-1}\left(\frac{1}{2}\right)$ is___________.

49. The principal value of $\tan^{-1}\sqrt{3}$ is___________.

50. The value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$ is___________.

51. The value of $\cos(\sin^{-1}x + \cos^{-1}x)$, $|x| \le 1$ is_______ .

52. The value of expression $\tan\left(\frac{\sin^{-1}x+\cos^{-1}x}{2}\right)$, when $x = \frac{\sqrt{3}}{2}$ is__________.

53. If $y = 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ for all x, then____< y <____.

54. The result $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ is true when value of xy is ______

55. The value of $\cot^{-1}(-x)$ for all $x \in R$ in terms of $\cot^{-1}x$ is ________.

MATRICES

CHAPTER – 3: MATRICES

MARKS WEIGHTAGE – 05 marks

NCERT Important Questions & Answers

1. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Ans:

Since, a matrix containing 18 elements can have any one of the following orders :

$$1 \times 18,\ 18 \times 1,\ 2 \times 9,\ 9 \times 2,\ 3 \times 6, 6 \times 3$$

Similarly, a matrix containing 5 elements can have order 1×5 or 5×1.

2. Construct a 3 × 4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

Ans:

(i) The order of given matrix is 3×4, so the required matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}, \text{ where } a_{ij} = \frac{1}{2}|-3i + j|$$

Putting the values in place of i and j, we will find all the elements of matrix A.

$$a_{11} = \frac{1}{2}|-3+1| = 1,\ a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2},\ a_{13} = \frac{1}{2}|-3+3| = 0$$

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2},\ a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2},\ a_{22} = \frac{1}{2}|-6+2| = 2$$

$$a_{23} = \frac{1}{2}|-6+3| = \frac{3}{2},\ a_{24} = \frac{1}{2}|-6+4| = 1,\ a_{31} = \frac{1}{2}|-9+1| = 4$$

$$a_{32} = \frac{1}{2}|-9+2| = \frac{7}{2},\ a_{33} = \frac{1}{2}|-9+3| = 3,\ a_{34} = \frac{1}{2}|-9+4| = \frac{5}{2}$$

Hence, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}_{3\times 4}$

(ii) Here, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$,where $a_{ij} = 2i - j$

$a_{11} = 2 - 1 = 1,$ $a_{12} = 2 - 2 = 0,$
$a_{13} = 2 - 3 = -1,$ $a_{14} = 2 - 4 = -2,$
$a_{21} = 4 - 1 = 3,$ $a_{22} = 4 - 2 = 2,$
$a_{23} = 4 - 3 = 1,$ $a_{24} = 4 - 4 = 0$
$a_{31} = 6 - 1 = 5,$ $a_{32} = 6 - 2 = 4,$
$a_{33} = 6 - 3 = 3$ and $a_{34} = 6 - 4 = 2$

Hence, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}_{3\times 4}$

3. Find the value of *a*, *b*, *c* and *d* from the equation: $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Ans:

Given that $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$a - b = -1$ …(i)

$2a - b = 0$ …(ii)

$2a + c = 5$ …(iii)

and $3c + d = 13$ …(iv)

Subtracting Eq.(i) from Eq.(ii), we get $a = 1$

Putting $a = 1$ in Eq. (i) and Eq. (iii), we get

$1 - b = -1$ and $2 + c = 5$

$\Rightarrow b = 2$ and $c = 3$

Substituting $c = 3$ in Eq. (iv), we obtain

$3 \times 3 + d = 13 \Rightarrow d = 13 - 9 = 4$

Hence, $a = 1, b = 2, c = 3$ and $d = 4$.

4. Find X and Y, if X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ **and X – Y =** $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$**.**

Ans:

$$(X+Y)+(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$Now, (X+Y)-(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

5. Find the values of *x* and *y* from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Ans:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

or $2x + 3 = 7$ and $2y - 4 = 14$

or $2x = 7 - 3$ and $2y = 18$

or $x = \frac{4}{2}$ and $y = \frac{18}{2}$

i.e. $x = 2$ and $y = 9$.

6. Find AB, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$**.**

Ans:

We have $AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Thus, if the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

7. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$**, then show that** $\mathbf{A^3 - 23A - 40\,I = O}$

Ans:

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$So, A^3 = A.A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$Now, A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

8. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$**, find the values of *x* and *y*.**

Ans:

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$$2x - y = 10 \quad \ldots(i)$$

$$\text{and } 3x + y = 5 \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$5x = 15 \Rightarrow x = 3$$

Substituting $x = 3$ in Eq. (i), we get

$$2 \times 3 - y = 10 \Rightarrow y = 6 - 10 = -4$$

9. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.

Ans:

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$$3x = x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$$

$$\text{and } 3y = 6 + x + y \Rightarrow 2y = 6 + x \Rightarrow y = \frac{6+x}{2}$$

Putting the value of x, we get

$$y = \frac{6+2}{2} = \frac{8}{2} = 4$$

Now, $3z = -1 + z + w$, $2z = -1 + w$

$$z = \frac{-1+w}{2} \quad \ldots(i)$$

$$\text{Now, } 3w = 2w + 3 \Rightarrow w = 3$$

Putting the value of w in Eq. (i), we get

$$z = \frac{-1+3}{2} = \frac{2}{2} = 1$$

Hence, the values of x, y, z and w are 2, 4, 1 and 3.

10. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F($x$) F($y$) = F($x$ + y).

Ans:

$$LHS = F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\sin y \cos x - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = RHS$$

11. Find A^2 – 5A + 6I, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Ans:

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

12. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Ans:

$$A^2 = A.A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

13. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Ans:

Given than $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$3k - 2 = 1 \Rightarrow k = 1$
$-2k = -2 \Rightarrow k = 1$
$4k = 4 \Rightarrow k = 1$
$-4 = -2k - 2 \Rightarrow k = 1$

Hence, $k = 1$

14. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ **and I is the identity matrix of order 2, show that**

I + A = (I – A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Ans:

Let $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$ *where* $x = \tan\frac{\alpha}{2}$

Now, $\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{1-x^2}{1+x^2}$ *and* $\cos\alpha = \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{2x}{1+x^2}$

$$RHS = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\right)\begin{bmatrix} \frac{1-x^2}{1+x^2} & -\frac{2x}{1+x^2} \\ \frac{2x}{1+x^2} & \frac{1-x^2}{1+x^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}\begin{bmatrix} \frac{1-x^2}{1+x^2} & -\frac{2x}{1+x^2} \\ \frac{2x}{1+x^2} & \frac{1-x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} \frac{1-x^2+2x^2}{1+x^2} & \frac{-2x+x(1-x^2)}{1+x^2} \\ \frac{-x(1-x^2)+2x}{1+x^2} & \frac{2x^2+1-x^2}{1+x^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+x^2}{1+x^2} & \frac{-2x+x-x^3}{1+x^2} \\ \frac{-x+x^3+2x}{1+x^2} & \frac{1+x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{-x-x^3}{1+x^2} \\ \frac{x^3+x}{1+x^2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-x(1+x^2)}{1+x^2} \\ \frac{x(x^2+1)}{1+x^2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}$$

$$LHS = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} = \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix} = RHS$$

15. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ **as the sum of a symmetric and a skew symmetric matrix.**

Ans:

$$B=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}\Rightarrow B'=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$$

$$Let\ \ P=\frac{1}{2}(B+B')=\frac{1}{2}\begin{bmatrix}4&-3&-3\\-3&6&2\\-3&2&-6\end{bmatrix}=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}$$

$$Now\ \ P'=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(B+B')$ is a symmetric matrix.

$$Also, let\ \ Q=\frac{1}{2}(B-B')=\frac{1}{2}\begin{bmatrix}0&-1&-5\\1&0&6\\5&-6&0\end{bmatrix}=\begin{bmatrix}0&-1/2&-5/2\\1/2&0&3\\5/2&-3&0\end{bmatrix}$$

$$Now\ \ Q'=\begin{bmatrix}0&1/2&5/2\\-1/2&0&-3\\-5/2&3&0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(B-B')$ is a skew symmetric matrix.

$$Now, P+Q=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}+\begin{bmatrix}0&-1/2&-5/2\\1/2&0&3\\5/2&-3&0\end{bmatrix}=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}=B$$

Thus, B is represented as the sum of a symmetric and a skew symmetric matrix.

16. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$(i)\begin{bmatrix}3&5\\1&-1\end{bmatrix}\qquad (ii)\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\qquad (iii)\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\qquad (iv)\begin{bmatrix}1&5\\-1&2\end{bmatrix}$$

Ans:

(i)

Let $A=\begin{bmatrix}3&5\\1&-1\end{bmatrix}, then\ \ A=P+Q$

$$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}6&6\\6&-2\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}3 & 5\\1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\5 & -1\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 4\\-4 & 0\end{bmatrix}=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}3 & 3\\3 & -1\end{bmatrix}+\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}=\begin{bmatrix}3 & 5\\1 & -1\end{bmatrix}=A$$

(ii)

Let $A=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}, then\ \ A=P+Q$

$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}12 & -4 & 4\\-4 & 6 & -2\\4 & -2 & 6\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}-\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}=A$$

(iii)

Let $A=\begin{bmatrix}3 & 3 & -1\\-2 & -2 & 1\\-4 & -5 & 2\end{bmatrix}, then\ \ A=P+Q$

$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}+\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}=P$$

Thus $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}-\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}=\begin{bmatrix}0&5/2&3/2\\-5/2&0&2\\-3/2&-2&0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0&-5/2&-3/2\\5/2&0&2\\3/2&-2&0\end{bmatrix}=-Q$$

Thus $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}+\begin{bmatrix}0&5/2&3/2\\-5/2&0&2\\-3/2&-2&0\end{bmatrix}=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}=A$$

(iv)

Let $A=\begin{bmatrix}1&5\\-1&2\end{bmatrix}, then\ \ A=P+Q$

where, $P=\frac{1}{2}(A+A')$ *and* $Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}1&5\\-1&2\end{bmatrix}+\begin{bmatrix}1&-1\\5&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}2&4\\4&4\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}1&2\\2&2\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}=P$$

Thus $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}1&5\\-1&2\end{bmatrix}-\begin{bmatrix}1&-1\\5&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0&6\\-6&0\end{bmatrix}=\begin{bmatrix}0&3\\-3&0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0&-3\\3&0\end{bmatrix}=-Q$$

Thus $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}=A$$

17. If A = $\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and A + A′ = I, find the value of α .

Ans:

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\Rightarrow A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$Now, A+A'=I$

$$\Rightarrow\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Comparing the corresponding elements of the above matrices, we have

$$2\cos\alpha=1\Rightarrow\cos\alpha=\frac{1}{2}=\cos\frac{\pi}{3}\Rightarrow\alpha=\frac{\pi}{3}$$

18. Show that the matrix *B′ AB* is symmetric or skew-symmetric according as *A* is symmetric or skew-symmetric.

Ans:

We suppose that A is a symmetric matrix, then $A' = A$

Consider $(B'AB)' = \{B'(AB)\}' = (AB)'(B')'$ $[\because (AB)' = B'A']$

$= B'A'$ (b) $[\because (B')' = B]$

$= B'(A'B) = B'(AB)$ $[\because A' = A]$

$\Rightarrow (B'AB)' = B'AB$

which shows that $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then, $A' = -A$

Consider $(B'AB)' = [B'(AB)]' = (AB)'(B')'$ $[\because (AB)' = B'A'$ and $(A')' = A]$

$= (B'A')B = B'(-A)B = -B'AB$ $[\because A' = -A]$

$\Rightarrow (B'AB)' = -B'AB$

which shows that $B'AB$ is a skew-symmetric matrix.

19. If *A* and *B* are symmetric matrices, prove that *AB* − *BA* is a skew-symmetric matrix.

Ans:

Here, A and $B$are symmetric matrices, then $A' = A$ and $B' = B$

Now, $(AB - BA)' = (AB)' - (BA)'$ $(\because (A - B)' = A' - B'$ and $(AB)' = B'A')$

$= B'A' - A'B' = BA - AB$ $(\because B' = B$and $A' = A)$

$= -(AB - BA)$

$\Rightarrow (AB - BA)' = -(AB - BA)$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

20. If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then prove that $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$, where *n* is any positive integer.

Ans:

We are required to prove that for all $n\in N$

$$P(n)=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$$

Let n = 1, then $P(1)=\begin{bmatrix}1+2(1) & -4(1)\\ 1 & 1-2(1)\end{bmatrix}=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ ....(i)

which is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k) = A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \text{........}(ii)$$

Let $n = k + 1$

$$P(k+1) = A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$Now, LHS = A^{k+1} = A^k A^1 = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+2k).3+(-4k).1 & (1+2k).(-4)+(-4k)(-1) \\ k.3+(1-2k).1 & k.(-4)+(1-2k)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ k+1 & -1-2k \end{bmatrix} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & -1-2(k+1) \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$whenever it is true for $n = k$. So, by principle of mathematical induction, it is true for all $n \in N$.

21. For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

Ans:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

Since Matrix multiplication is associative, therefore $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0+4+0 \\ 0+0+x \\ 0+0+2x \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ x \\ 2x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 4+2x+2x \end{bmatrix} = O \Rightarrow \begin{bmatrix} 4+4x \end{bmatrix} = O$$

$$\Rightarrow 4+4x = 0 \Rightarrow 4x = -4 \Rightarrow x = -1$$

22. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Ans:

Given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=O$$

23. Find *x*, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=O$?

Ans:

Given that $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=O$

Since Matrix multiplication is associative, therefore $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} x+0+2 \\ 0+8+1 \\ 2x+0+3 \end{bmatrix}=O$

$$\Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} x+2 \\ 9 \\ 2x+3 \end{bmatrix}=O$$

$$\Rightarrow \left[x(x+2)+(-5).9+(-1)(2x+3)\right]=O$$

$$\Rightarrow \left[x^2-48\right]=O \Rightarrow x^2-48=0 \Rightarrow x^2=48$$

$$\Rightarrow x=\pm\sqrt{48}=\pm 4\sqrt{3}$$

24. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

Ans:

Given that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

The matrix given on the RHS of the equation is a 2 × 3matrix and the one given on the LHS of the equation is as a 2 × 3 matrix. Therefore, *X* has to be a 2 × 2 matrix.

Now, let $X=\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$\begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix}=\begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have

$a + 4c = -7$, $2a + 5c = -8$, $3a + 6c = -9$

$b + 4d = 2$, $2b + 5d = 4$, $3b + 6d = 6$

Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$

$2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$

$\Rightarrow -3c = 6 \Rightarrow c = -2$

$\therefore\ a = -7 - 4(-2) = -7 + 8 = 1$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$ and $2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$

$\therefore\ -3d = 0 \Rightarrow d = 0$

$\therefore\ b = 2 - 4(0) = 2$

Thus, $a = 1$, $b = 2$, $c = -2$, $d = 0$

Hence, the required matrix *X* is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

25. If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$**, then prove that** $A^n=\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n\in N$

Ans:

We shall prove the result by using principle of mathematical induction.

We have P(n) : If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

Let n = 1, then $P(1)=A^1=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Therefore, the result is true for n = 1.

Let the result be true for $n = k$. So

$P(k)=A^k=\begin{bmatrix}\cos k\theta & \sin k\theta\\ -\sin k\theta & \cos k\theta\end{bmatrix}$

Now, we prove that the result holds for $n = k + 1$

i.e. $P(k+1)=A^{k+1}=\begin{bmatrix}\cos(k+1)\theta & \sin(k+1)\theta\\ -\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$

Now, $P(k+1)=A^{k+1}=A^k.A=\begin{bmatrix}\cos k\theta & \sin k\theta\\ -\sin k\theta & \cos k\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos\theta\cos k\theta-\sin\theta\sin k\theta & \cos\theta\sin k\theta+\sin\theta\cos k\theta\\ -\sin\theta\cos k\theta+\cos\theta\sin k\theta & -\sin\theta\sin k\theta+\cos\theta\cos k\theta\end{bmatrix}$

$=\begin{bmatrix}\cos(k\theta+\theta) & \sin(k\theta+\theta)\\ -\sin(k\theta+\theta) & \cos(k\theta+\theta)\end{bmatrix}=\begin{bmatrix}\cos(k+1)\theta & \sin(k+1)\theta\\ -\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$

Therefore, the result is true for $n = k + 1$. Thus by principle of mathematical induction, we have

$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

26. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}, C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that CD – AB = O.

Ans:

Since A, B, C are all square matrices of order 2, and CD – AB is well defined, D must be a square matrix of order 2.

$Let\ \ D=\begin{bmatrix}a & b\\ c & d\end{bmatrix},\ then\ \ CD-AB=0\Rightarrow\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}-\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}=O$

$\Rightarrow\begin{bmatrix}2a+5c & 2b+5d\\ 3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2a+5c-3 & 2b+5d\\ 3a+8c-43 & 3b+8d-22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

By equality of matrices, we get

$2a + 5c - 3 = 0$ ... (1)

$3a + 8c - 43 = 0$ ... (2)

$2b + 5d = 0$ ... (3)

and $3b + 8d - 22 = 0$ ... (4)

Solving (1) and (2), we get $a = -191$, $c = 77$. Solving (3) and (4), we get $b = -110$, $d = 44$.

Therefore $D=\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$

CHAPTER – 3: MATRICES

MARKS WEIGHTAGE – 05 marks

Previous Years Board Exam (Important Questions & Answers)

1. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ write the value of $a - 2b$.

Ans:

Give that $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$

On equating, we get

$a + 4 = 2a + 2$, $3b = b + 2$, $a - 8b = -6$

$\Rightarrow a = 2, b = 1$

Now the value of $a - 2b = 2 - (2 \times 1) = 2 - 2 = 0$

2. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

Ans:

$7A - (I + A)^3 = 7A - \{I^3 + 3I^2A + 3I.A^2 + A^3\}$

$= 7A - \{I + 3A + 3A + A^2A\}$ $\qquad [\because I^3 = I^2 = I, A^2 = A]$

$= 7A - \{I + 6A + A^2\} = 7A - \{I + 6A + A\}$

$= 7A - \{I + 7A\} = 7A - I - 7A = -I$

3. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

Ans:

Given that $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

Equating, we get

$x - y = -1 \ldots (i)$

$2x - y = 0 \ldots (ii)$

$z = 4, w = 5$

$(ii) - (i) \Rightarrow 2x - y - x + y = 0 + 1$

$\Rightarrow x = 1$ and $y = 2$

$\therefore x + y = 2 + 1 = 3.$

4. Solve the following matrix equation for x : $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

Ans:

Given that $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

5. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find ($x - y$).

Ans:

Given that $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

Equating we get $8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow y = -8$ and $x = 2$

$\Rightarrow x - y = 2 + 8 = 10$

6. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

Ans:

A will be skew symmetric matrix if

$A = -A'$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

Equating, we get $x = 2$

7. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k.

Ans:

Given $A^2 = kA$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow k = 2$

8. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Ans:

Given that $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Equating the corresponding elements we get.

$a - b = -1 \ldots (i)$

$2a + c = 5 \ldots (ii)$

$2a - b = 0 \ldots (iii)$

$3c + d = 13 \ldots (iv)$

From (iii) $2a = b$

$\Rightarrow a = \frac{b}{2}$

Putting in (i) we get $\frac{b}{2} - b = -1$

$\Rightarrow -\frac{b}{2} = -1 \Rightarrow b = 2$

$\therefore\ a = 1$

(ii) $c = 5 - 2 \times 1 = 5 - 2 = 3$

(iv) $d = 13 - 3 \times (3) = 13 - 9 = 4$

i.e. $a = 1$, $b = 2$, $c = 3$, $d = 4$

9. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Ans:

Given that $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

10. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

Ans:

$(I + A)^2 - 3A = I^2 + A^2 + 2A - 3A$

$= I^2 + A^2 - A$

$= I^2 + A - A\ [\because A^2 = A]$

$= I^2 = I \cdot I = I$

11. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, write the value of x.

Ans:

Given that $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Equating the corresponding elements we get.

$2x - y = 10$ ...(i)

$3x + y = 5$ ...(ii)

Adding (i) and (ii), we get $2x - y + 3x + y = 10 + 5$

$\Rightarrow 5x = 15 \Rightarrow x = 3.$

12. Find the value of $x + y$ from the following equation: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Ans:

Given that $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding element we get

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow x = \frac{7-3}{2}$ *and* $y = \frac{14+4}{2}$

$\Rightarrow x = 2$ and $y = 9$

$\therefore\ x + y = 2 + 9 = 11$

13. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ ***and*** $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then find $A^T - B^T$.**

Ans:

Given that $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

14. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$**, write the value of x**

Ans:

Given that $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating the corresponding elements, we get

$x = 13$

15. Simplify: $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Ans:

$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

16. Write the values of $x - y + z$ from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans:

By definition of equality of matrices, we have

$x + y + z = 9$ ... (i)

$x + z = 5$ .... (ii)

$y + z = 7$ ... (iii)

(i) – (ii) $\Rightarrow x + y + z - x - z = 9 - 5$

$\Rightarrow y = 4$ ... (iv)

(ii) – (iv) $\Rightarrow x - y + z = 5 - 4$

$\Rightarrow x - y + z = 1$

17. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$**, then find the value of y.**

Ans:

Given that $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$

By definition of equality of matrices, we have

$y + 2x = 7$

$-x = -2 \Rightarrow x = 2$

$\therefore y + 2(2) = 7 \Rightarrow y = 3$

OBJECTIVE TYPE QUESTIONS (1 MARK)

1. If a matrix has 6 elements, then number of possible orders of the matrix can be
(a) 2 (b) 4 (c) 3 (d) 6

2. If A and B are square matrices of the same order, then (A + B) (A – B) is equal to
(a) $A^2 - B^2$ (b) $A^2 - BA - AB - B^2$ (c) $A^2 - B^2 + BA - AB$ (d) $A^2 - BA + B^2 + AB$

3. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then
(a) only AB is defined (b) only BA is defined
(c) AB and BA both are defined (d) AB and BA both are not defined.

4. The matrix $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a
(a) scalar matrix (b) diagonal matrix (c) unit matrix (d) square matrix

5. If A and B are symmetric matrices of the same order, then $(AB' - BA')$ is a
(a) Skew symmetric matrix (b) Null matrix (c) Symmetric matrix (d) None of these

6. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a
(a) square matrix (b) diagonal matrix (c) unit matrix (d) none

7. If $A = [a_{ij}]$ is a 2×3 matrix, such that $aij = \frac{(-i+2j)^2}{5}$, then a_{23} is
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{9}{5}$ (d) $\frac{16}{5}$

8. If A = diag(3, –1), then matrix A is
(a) $\begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$

9. Total number of possible matrices of order 2×3 with each entry 1 or 0 is
(a) 6 (b) 36 (c) 32 (d) 64

10. If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is
(a) I (b) 2A (c) 3I (d) A

11. If matrices A and B are inverse of each other then
(a) AB = BA (b) AB = BA = I (c) AB = BA = O (d) AB = O, BA = I

12. The diagonal elements of a skew symmetric matrix are
(a) all zeroes (b) are all equal to some scalar $k(\neq 0)$
(c) can be any number (d) none of these

13. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = A′ then

(a) x = 0, y = 5 (b) x = y (c) x + y = 5 (d) x – y = 5

14. If a matrix A is both symmetric and skew symmetric then matrix A is

(a) a scalar matrix (b) a diagonal matrix

(c) a zero matrix of order n × n (d) a rectangular matrix.

15. If F(x) = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ then F(x) F(y) is equal to

(a) F(x) (b) F(xy) (c) F(x + y) (d) F(x – y)

16. If A = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then A^6 is equal to

(a) zero matrix (b) A (c) I (d) none of these

17. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 5A - 7I$ is

(a) a zero matrix (b) an identity matrix

(c) diagonal matrix (d) none of these

18. The matrix A satisfies the equation $\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then matrix A is

(a) $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

19. If A = $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, then A^2 is

(a) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 \\ 4 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

20. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

21. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x + y is

(a) x = 3, y = 1 (b) x = 2, y = 3 (c) x = 2, y = 4 (d) x = 3, y = 3

22. If A = $\frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$, B = $-\frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then A – B is equal to

(a) I (b) O (c) 2I (d) $\frac{1}{2}I$

23. If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively, and $m = n$, then the order of matrix $(5A - 2B)$ is
(a) $m \times 3$ (b) 3×3 (c) $m \times n$ (d) $3 \times n$

24. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to
(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

25. If matrix $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = 1$ if $i \neq j$ and 0 if $i = j$ then A^2 is equal to
(a) I (b) A (c) 0 (d) None of these

26. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a
(a) identity matrix (b) symmetric matrix (c) skew symmetric matrix (d) none of these

27. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a
(a) diagonal matrix (b) symmetric matrix (c) skew symmetric matrix (d) scalar matrix

28. If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is
(a) $m \times m$ (b) $n \times n$ (c) $n \times m$ (d) $m \times n$

29. If A and B are matrices of same order, then $(AB' - BA')$ is a
(a) skew symmetric matrix (b) null matrix (c) symmetric matrix (d) unit matrix

30. On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation
$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have :
(a) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 0 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

31. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to
(a) A (b) I – A (c) I + A (d) 3A

32. For any two matrices A and B, we have
(a) AB = BA (b) $AB \neq BA$ (c) AB = O (d) None of the above

33. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation:
$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, we have :

(a) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 2 \\ -5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

34. If A and B are two skew symmetric matrices of same order, then AB is symmetric matrix if ________.

35. If A and B are matrices of same order, then (3A –2B)′ is equal to ________.

36. Addition of matrices is defined if order of the matrices is ________

37. _________ matrix is both symmetric and skew symmetric matrix.

38. Sum of two skew symmetric matrices is always _________ matrix.

39. The negative of a matrix is obtained by multiplying it by _________.

40. The product of any matrix by the scalar _________ is the null matrix.

41. A matrix which is not a square matrix is called a _________ matrix.

42. Matrix multiplication is _________ over addition.

43. If A is a symmetric matrix, then A^3 is a _________ matrix.

44. If A is a skew symmetric matrix, then A^2 is a _________.

45. If A and B are square matrices of the same order, then
(i) (AB)′ = _________.
(ii) (kA)′ = _________. (k is any scalar)
(iii) [k (A – B)]′ = _________.

46. If A is skew symmetric, then kA is a _________. (k is any scalar)

47. If A and B are symmetric matrices, then
(i) AB – BA is a _________.
(ii) BA – 2AB is a _________.

48. If A is symmetric matrix, then B′AB is _________.

49. If A and B are symmetric matrices of same order, then AB is symmetric if and only if _________.

50. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeros in one or more, then A^{-1} _________.

DETERMINANTS

CHAPTER – 4: DETERMINANTS

MARKS WEIGHTAGE – 05 marks

NCERT Important Questions & Answers

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then find the value of *x*.

Ans:

Given that $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

On expanding both determinants, we get

$x \times x - 18 \times 2 = 6 \times 6 - 18 \times 2 \Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0 \Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

2. Find values of *k* if area of triangle is 4 sq. units and vertices are (i) (*k*, 0), (4, 0), (0, 2) (ii) (–2, 0), (0, 4), (0, *k*)

Ans:

(i) We have Area of triangle = $\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$

$\Rightarrow |\, k(0 - 2) + 1(8 - 0)| = 8$

$\Rightarrow k(0 - 2) + 1(8 - 0) = \pm 8$

On taking positive sign $-2k + 8 = 8$

$\Rightarrow - 2k = 0$

$\Rightarrow k = 0$

On taking negative sign $-2k + 8 = - 8$

$\Rightarrow - 2k = - 16$

$\Rightarrow k = 8$

$\Rightarrow k = 0, 8$

(ii) We have Area of triangle = $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$

$\Rightarrow | - 2(4 - k) + 1(0 - 0)| = 8$

$\Rightarrow - 2(4 - k) + 1(0 - 0) = \pm 8$

$\Rightarrow [- 8 + 2k] = \pm 8$

On taking positive sign, $2k - 8 = 8 \Rightarrow 2k = 16 \Rightarrow k = 8$

On taking negative sign, $2k - 8 = - 8 \Rightarrow 2k = 0 \Rightarrow k = 0$

$\Rightarrow k = 0, 8$

3. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (*k*, 4). Then find the value of *k*.

Ans:

We have Area of triangle = $\frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35$

$\Rightarrow |2\,(4 - 4) + 6(5 - k) + 1(20 - 4k)| = 70$

$\Rightarrow 2\,(4 - 4) + 6\,(5 - k) + 1(20 - 4k) = \pm 70$

$\Rightarrow 30 - 6k + 20 - 4k = \pm 70$

On taking positive sign, $- 10k + 50 = 70$

$\Rightarrow -10k = 20 \Rightarrow k = -2$
On taking negative sign, $-10k + 50 = -70$
$\Rightarrow -10k = -120 \Rightarrow k = 12$
$\therefore k = 12, -2$

4. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Ans:

Given that $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Cofactors of the elements of second row

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9-16) = 7$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = (15-8) = 7$

and $A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$

Now, expansion of Δ using cofactors of elements of second row is given by
$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$
$= 2 \times 7 + 0 \times 7 + 1(-7) = 14 - 7 = 7$

5. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$**, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.**

Ans:

Given that A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, $A^2 - 5A + 7I = O$

$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-2 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore A^2 - 5A + 7I = O$

$\because |A| = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} = 6 + 1 = 7 \neq 0$

$\therefore A^{-1}$ exists.

Now, $A.A - 5A = -7I$
Multiplying by A^{-1} on both sides, we get
$A.A\,(A^{-1}) - 5A(A^{-1}) = -7I(A^{-1})$
$\Rightarrow AI - 5I = -7A^{-1}$ (using $AA^{-1} = I$ and $IA^{-1} = A^{-1}$)

$$A^{-1} = -\frac{1}{7}(A-5I) = \frac{1}{7}(5I-A) = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

6. For the matrix A = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, find the numbers *a* and *b* such that A^2 + *a*A + *b*I = O.

Ans:

Given that A = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Now, $A^2 + aA + bI = O$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

If two matrices are equal, then their corresponding elements are equal.

$\Rightarrow$ 11 + 3a + b = 0 …(i)

8 + 2a = 0 …(ii)

4 + a = 0 …(iii)

and 3 + a + b = 0 …(iv)

Solving Eqs. (iii) and (iv), we get 4 + a = 0

$\Rightarrow a = -4$

and 3 + a + b = 0

$\Rightarrow 3 - 4 + b = 0 \Rightarrow b = 1$

Thus, $a = -4$ and $b = 1$

7. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, Show that A^3– 6A^2 + 5A + 11 I = O. Hence, find A^{-1}.

Ans:

Given that A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$and \quad A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3-6A^2+5A+11I$

$$=\begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}-6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}+5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}+11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}-\begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}+\begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix}+\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$=\begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=O$$

$$\because |A|=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}=1(6-3)-1(3+6)+1(-1-4)=3-9-5=-11\neq 0$$

$\therefore A^{-1}$ *exist*

$Now, A^3-6A^2+5A+11I=O$

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+5(AA^{-1})+11(IA^{-1})=O$

$\Rightarrow AAI-6AI+5I+11A^{-1}=O$

$\Rightarrow A^2-6A+5I=-11A^{-1}$

$$\Rightarrow A^{-1}=-\frac{1}{11}(A^2-6A+5I)$$

$$\Rightarrow A^{-1}=\frac{1}{11}(-A^2+6A-5I)$$

$$\Rightarrow A^{-1}=\frac{1}{11}\left(\begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix}+6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}-5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1}=\frac{1}{11}\left(\begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix}+\begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}-\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1}=\frac{1}{11}\begin{bmatrix} -4+6-5 & -2+6-0 & -1+6-0 \\ 3+6-0 & -8+12-5 & 14-18-0 \\ -7+12-0 & 3-6-0 & -14+18-5 \end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

8. Solve system of linear equations, using matrix method,

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ and \ \ B = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{vmatrix} = 2(20+6) - 1(-10-0) + 1(6-0)$$

$= 52 + 10 + 6 = 68 \neq 0$

Thus, A is non-singular, Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$.

Cofactors of A are

$A_{11} = 20 + 6 = 26,$

$A_{12} = -(-10 + 0) = 10,$

$A_{13} = 6 + 0 = 6$

$A_{21} = -(-5 - 3) = 8,$

$A_{22} = -10 - 0 = -10,$

$A_{23} = -(6 - 0) = -6$

$A_{31} = (-2 + 4) = 2,$

$A_{32} = -(-4 - 2) = 6,$

$A_{33} = -8 - 2 = -10$

$$adj(A) = \begin{bmatrix} 26 & 10 & 6 \\ 8 & -10 & -6 \\ 2 & 6 & -10 \end{bmatrix}^T = \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{68}\begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 26+24+18 \\ 10-30+54 \\ 6-18-90 \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 68 \\ 34 \\ -102 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{-3}{2} \end{bmatrix}$$

$$Hence, \ x = 1, \ y = \frac{1}{2} \ and \ \ z = \frac{-3}{2}$$

9. Solve system of linear equations, using matrix method,

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ and \ B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1 + 3) - (-1)(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$$

Thus, A is non-singular, Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$.

Cofactors of A are

$A_{11} = 1 + 3 = 4$,

$A_{12} = -(2 + 3) = -5$,

$A_{13} = 2 - 1 = 1$,

$A_{21} = -(-1 - 1) = 2$,

$A_{22} = 1 - 1 = 0$,

$A_{23} = -(1 + 1) = -2$,

$A_{31} = 3 - 1 = 2$,

$A_{32} = -(-3 - 2) = 5$,

$A_{33} = 1 + 2 = 3$

$$adj(A) = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$ and $z = 1$.

10. Solve system of linear equations, using matrix method,

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ and \ B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3\ (-2 - 3) + 3\ (-1 + 6)$$

$= 10 + 15 + 15 = 40 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists. Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 4 + 1 = 5$,

$A_{12} = -(-2 - 3) = 5$,

$A_{13} = (-1 + 6) = 5$,

$A_{21} = -(-6 + 3) = 3$,

$A_{22} = (-4 - 9) = -13$,

$A_{23} = -(-2 - 9) = 11$,

$A_{31} = 3 + 6 = 9$,

$A_{32} = -(2 - 3) = 1$,

$A_{33} = -4 - 3 = -7$

$$adj(A) = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}^T = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = -1$.

11. Solve system of linear equations, using matrix method,

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix} = 1(12 - 5) - (-1)\ (9 + 10) + 2\ (-3 - 8)$$

$= 7 + 19 - 22 = 4 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 12 - 5 = 7$,

$A_{12} = -(9 + 10) = -19,$
$A_{13} = -3 - 8 = -11,$
$A_{21} = -(-3 + 2) = 1,$
$A_{22} = 3 - 4 = -1,$
$A_{23} = -(-1 + 2) = -1,$
$A_{31} = 5 - 8 = -3,$
$A_{32} = -(-5 - 6) = 11,$
$A_{33} = 4 + 3 = 7$

$$adj(A) = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2$, $y = 1$ and $z = 3$.

12. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ **find A^{-1}. Using A^{-1}, Solve system of linear equations:**

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

Ans:
The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4 + 4) - (-3)(-6 + 4) + 5(3 - 2)$$

$= 0 - 6 + 5 = -1 \neq 0$
Thus, A is non-singular. Therefore, its inverse exists.
Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$
Cofactors of A are
$A_{11} = -4 + 4 = 0,$
$A_{12} = -(-6 + 4) = 2,$
$A_{13} = 3 - 2 = 1,$
$A_{21} = -(6 - 5) = -1,$
$A_{22} = -4 - 5 = -9,$
$A_{23} = -(2 + 3) = -5,$
$A_{31} = (12 - 10) = 2,$
$A_{32} = -(-8 - 15) = 23,$

$A_{33} = 4 + 9 = 13$

$$adj(A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = 3$.

13. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Ans:

Let the prices (per kg) of onion, wheat and rice be Rs. x, Rs. y and Rs. z, respectively then

$4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12 - 12) - 3(6 - 36) + 2(4 - 24)$$

$= 0 + 90 - 40 = 50 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists. Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are,

$A_{11} = 12 - 12 = 0$,

$A_{12} = -(6 - 36) = 30$,

$A_{13} = 4 - 24 = -20$,

$A_{21} = -(9 - 4) = -5$,

$A_{22} = 12 - 12 = 0$,

$A_{23} = -(8 - 18) = 10$,

$A_{31} = (18 - 8) = 10$,

$A_{32} = -(24 - 4) = -20$,

$A_{33} = 16 - 6 = 10$

$$adj(A) = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}^T = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}\begin{bmatrix}60\\ 90\\ 70\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0-450+700\\ 1800+0-1400\\ -1200+900+700\end{bmatrix}=\frac{1}{50}\begin{bmatrix}250\\ 400\\ 400\end{bmatrix}=\begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$$

$\therefore$ $x = 5, y = 8$ and $z = 8$.

Hence, price of onion per kg is Rs. 5, price of wheat per kg is Rs. 8 and that of rice per kg is Rs. 8.

14. Solve the system of equations:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Ans:

Let $\frac{1}{x}=p,\ \frac{1}{y}=q\ \ and\ \ \frac{1}{z}=r$, then the given equations become

$2p + 3q + 10r = 4,\ 4p - 6q + 5r = 1,\ 6p + 9q - 20r = 2$

This system can be written as $AX = B$, where

$$A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}, X=\begin{bmatrix}p\\ q\\ r\end{bmatrix}\ \ and\ \ B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{vmatrix}=2(120-45)-3(-80-30)+10(36+36)$$

$= 150 + 330 + 720 = 1200 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the above system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 120 - 45 = 75,$

$A_{12} = -(-80 - 30) = 110,$

$A_{13} = (36 + 36) = 72,$

$A_{21} = -(-60 - 90) = 150,$

$A_{22} = (-40 - 60) = -100,$

$A_{23} = -(18 - 18) = 0,$

$A_{31} = 15 + 60 = 75,$

$A_{32} = -(10 - 40) = 30,$

$A_{33} = -12 - 12 = -24$

$$adj(A)=\begin{bmatrix}75 & 110 & 72\\ 150 & -100 & 0\\ 75 & 30 & -24\end{bmatrix}^T=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\Rightarrow p = \frac{1}{2}, q = \frac{1}{3}, r = \frac{1}{5}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$\Rightarrow x = 2$, $y = 3$ and $z = 5$.

15. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

Ans:

Given that $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = AA = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$Hence,\ A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$$Now,\ A^2 - 4A + I = O$$

$$\Rightarrow AA - 4A = -I$$

$\Rightarrow AA(A^{-1}) - 4AA^{-1} = -IA^{-1}$ (Post multiplying by A^{-1} because $|A| \neq 0$)

$$\Rightarrow A(AA^{-1}) - 4I = -A^{-1}$$

$$\Rightarrow AI - 4I = -A^{-1}$$

$$\Rightarrow A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

16. Solve the following system of equations by matrix method.

$3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$

Ans:

The system of equation can be written as $AX = B$, where

$$A=\begin{bmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \ and \ B=\begin{bmatrix}8\\ 1\\ 4\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{vmatrix}$$

$= 3 (2 – 3) + 2(4 + 4) + 3 (– 6 – 4) = – 17 \neq 0$

Hence, A is nonsingular and so its inverse exists. Now

$A_{11} = –1$, $A_{12} = – 8$, $A_{13} = –10$

$A_{21} = –5$, $A_{22} = – 6$, $A_{23} = 1$

$A_{31} = –1$, $A_{32} = 9$, $A_{33} = 7$

$$adj(A)=\begin{bmatrix}-1 & -8 & 10\\ -5 & -6 & 1\\ -1 & 9 & 7\end{bmatrix}^T=\begin{bmatrix}-1 & -5 & -1\\ -8 & -6 & 9\\ -10 & 1 & 7\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=-\frac{1}{17}\begin{bmatrix}-1 & -5 & -1\\ -8 & -6 & 9\\ -10 & 1 & 7\end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=-\frac{1}{17}\begin{bmatrix}-1 & -5 & -1\\ -8 & -6 & 9\\ -10 & 1 & 7\end{bmatrix}\begin{bmatrix}8\\ 1\\ 4\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=-\frac{1}{17}\begin{bmatrix}-17\\ -34\\ -51\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

Hence $x = 1$, $y = 2$ and $z = 3$.

17. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x – y + 2z = 1$
$2y – 3z = 1$
$3x – 2y + 4z = 2$

Ans:

Consider the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$

$$=\begin{bmatrix}-2-9+12 & 0-2+2 & 1+3-4\\ 0+18-18 & 0+4-3 & 0-6+6\\ -6-18+24 & 0-4+4 & 3+6-8\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$Hence, \begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}^{-1}=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$$

Now, given system of equations can be written, in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

Hence $x = 0$, $y = 5$ and $z = 3$

CHAPTER – 3: DETERMINANTS

MARKS WEIGHTAGE – 05 marks

Previous Years Board Exam (Important Questions & Answers)

1. Let A be a square matrix of order 3 × 3. Write the value of $|2A|$, where $|A| = 4$.
Ans:
Since $|2A| = 2^n|A|$ where n is order of matrix A.
Here $|A| = 4$ and $n = 3$
$\therefore\ |2A| = 2^3 \times 4 = 32$

2. Write the value of the following determinant: $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Ans:
Given that $\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$
Applying $R_1 \to R_1 - 6R_3$
$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$ (Since R_1 is zero)

3. If A is a square matrix and $|A| = 2$, then write the value of $|AA'|$, where A' is the transpose of matrix A.
Ans:
$|AA|' = |A|.|A'| = |A|.|A| = |A|^2 = 2 \text{ x } 2 = 4$.
[since, $|AB|=|A|.|B|$ and $|A|=|A'|$, where A and B are square matrices.]

4. If A is a 3 × 3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k.
Ans:
Here, $|3A| = k|A|$
$\Rightarrow 3^3|A| = k|A|$ [$\because$ $|kA| = kn|A|$ where n is order of A]
$\Rightarrow 27|A| = k|A|$
$\Rightarrow k = 27$

5. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Ans:
$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}\pi = (a+ib)(a-ib)-(c+id)(-c+id)$
$= (a+ib)(a-ib)+(c+id)(c-id)$
$= a^2 - i^2b^2 + c^2 - i^2d^2$
$= a^2 + b^2 + c^2 + d^2$

6. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, find the value of x.
Ans:

Given that $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$

$\Rightarrow$ $4x + 8 - 3x - 15 = 3$

$\Rightarrow x - 7 = 3$

$\Rightarrow x = 10$

7. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

Ans:

Minor of a_{23} = $\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$ = 10 – 3 = 7.

8. Evaluate: $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

Ans:

Expanding the determinant, we get

cos 15° . cos 75° - sin 15° . sin 75°

= cos (15° + 75°) = cos 90° = 0

[since cos (A + B) = cos A. cos B – sinA . sin B]

9. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Ans:

Given determinant |A|= $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

$= 3x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 0 (\because R_1 = R_3)$

10. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 2,200. School Q wants to spend Rs. 3,100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs. 1,200, using matrices, find the award money for each value. Apart from these three values, suggest one more value that should be considered for award.

Ans:

According to question,

$3x + 2y + z = 2200$

$4x + y + 3z = 3100$

$x + y + z = 1200$

The above system of equation may be written in matrix form as $AX = B$

$$A=\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}2200\\3100\\1200\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{vmatrix}=3(1-3)-2(4-3)+1(4-1)=-6-2+3=-5\neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = (1 - 3) = -2$,

$A_{12} = -(4 - 3) = -1$,

$A_{13} = (4 - 1) = 3$,

$A_{21} = -(2 - 1) = -1$,

$A_{22} = (3 - 1) = 2$,

$A_{23} = -(3 - 2) = -1$

$A_{31} = (6 - 1) = 5$,

$A_{32} = -(9 - 4) = -5$,

$A_{33} = (3 - 8) = -5$

$$adj(A)=\begin{bmatrix}-2 & -1 & 3\\-1 & 2 & -1\\5 & -5 & -5\end{bmatrix}^T=\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{-5}\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}=\frac{1}{5}\begin{bmatrix}2 & 1 & -5\\1 & -2 & 5\\-3 & 1 & 5\end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}2 & 1 & -5\\1 & -2 & 5\\-3 & 1 & 5\end{bmatrix}\begin{bmatrix}2200\\3100\\1200\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}4400+3100-6000\\2200-6200+6000\\-6600+3100+6000\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1500\\2000\\2500\end{bmatrix}=\begin{bmatrix}300\\400\\500\end{bmatrix}$$

$\Rightarrow$ $x = 300$, $y = 400$, $z = 500$

i.e., Rs. 300 for tolerance, Rs. 400 for kindness and Rs. 500 for leadership are awarded.

One more value like punctuality, honesty etc may be awarded.

11. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

Ans:

Let no. of students in Ist, 2nd and 3rd group to *x*, *y*, *z* respectively.

From the statement we have

$$x + y + z = 10$$
$$2x + y = 13$$
$$x + y - 4z = 0$$

The above system of linear equations may be written in matrix form as $AX = B$ where

$$A=\begin{bmatrix}1&1&1\\2&1&0\\1&1&-4\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}10\\13\\0\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}1&1&1\\2&1&0\\1&1&-4\end{vmatrix}=1(-4-0)-1(-8-0)+1(2-1)=-4+8+1=5\neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -4 - 0 = -4$

$A_{12} = -(-8-0) = 8$

$A_{13} = 2 - 1 = 1$

$A_{21} = -(-4-1) = 5$

$A_{22} = -4 - 1 = -5$

$A_{23} = -(1-1) = 0$

$A_{31} = 0 - 1 = -1$

$A_{32} = -(0-2) = 2$

$A_{33} = 1 - 2 = -1$

$$adj(A)=\begin{bmatrix}-4&8&1\\5&-5&0\\-1&2&-1\end{bmatrix}^T=\begin{bmatrix}-4&5&-1\\8&-5&2\\1&0&-1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{5}\begin{bmatrix}-4&5&-1\\8&-5&2\\1&0&-1\end{bmatrix}$$

Now, $X=A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}-4&5&-1\\8&-5&2\\1&0&-1\end{bmatrix}\begin{bmatrix}10\\13\\0\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}-40+65\\80-65\\10\end{bmatrix}=\frac{1}{5}\begin{bmatrix}25\\15\\10\end{bmatrix}=\begin{bmatrix}5\\3\\2\end{bmatrix}$$

$\Rightarrow x = 5, y = 3, z = 2$

12. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Ans:

According to question

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

The above system of linear equation can be written in matrix form as $AX = B$ where

$$A=\begin{bmatrix}1&1&1\\2&3&3\\1&-2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}12\\33\\0\end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

= 1 (3 + 6) –1 (2–3) + 1 (– 4 –3) = 9 + 1 – 7 = 3

$\therefore$ A^{-1} exists.

$A_{11} = 9$, $A_{12} = 1$, $A_{13} = -7$

$A_{21} = -3$, $A_{22} = 0$, $A_{23} = 3$

$A_{31} = 0$, $A_{32} = -1$, $A_{33} = 1$

$$adj(A) = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108-99 \\ 12+0+0 \\ -84+99 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow x = 3, y = 4, z = 5$

No. of awards for honesty = 3

No. of awards for helping others = 4

No. of awards for supervising = 5.

The persons, who work in the field of health and hygiene should also be awarded.

13. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. **6,000. Three times the award money for Hardwork added to that given for honesty amounts to ` 11,000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hardwork, suggest one more value which the school must include for awards.**

Ans:

Let x, y and z be the awarded money for honesty, Regularity and hardwork.

From the statement

$$x + y + z = 6000 \ldots(i)$$
$$x + 3z = 11000 \ldots(ii)$$
$$x + z = 2y \Rightarrow x - 2y + z = 0 \ldots(iii)$$

The above system of three equations may be written in matrix form as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 1(0+6) - 1(1-3) + 1(-2-0) = 6 + 2 - 2 = 6 \neq 0$$

Hence A^{-1} exist
If Aij is co-factor of aij then
$A_{11} = 0 + 6 = 6$
$A_{12} = -(1-3) = 2$;
$A_{13} = (-2-0) = -2$
$A_{21} = -(1+2) = -3$
$A_{22} = 0$
$A_{23} = (-2-1) = -3$
$A_{31} = 3 - 0 = 3$
$A_{32} = -(3-1) = -2$;
$A_{33} = 0 - 1 = -1$

$$adj(A) = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 + 0 \\ -12000 + 33000 + 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$\Rightarrow x = 500, y = 2000, z = 3500$
Except above three values, school must include discipline for award as discipline has great importance in student's life.

14. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Ans:

Given that $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2) - (x-1)(x-3) = 12 + 1$
$\Rightarrow x^2 + 2x + x + 2 - x^2 + 3x + x - 3 = 13$
$\Rightarrow 7x - 1 = 13$
$\Rightarrow 7x = 14$
$\Rightarrow x = 2$

15. Using matrices, solve the following system of equations:

$x - y + z = 4$; $2x + y - 3z = 0$; $x + y + z = 2$

Ans:

Given equations
$x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 2$
We can write this system of equations as AX = B where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ and \ \ B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

= 1 (1 + 3) - (- 1) (2 + 3) + 1 (2 - 1) = 4 + 5 + 1 = 10

$\therefore$ A^{-1} exists.

$A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$adj(A) = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

The required solution is

$\therefore$ $x = 2, y = -1, z = 1$

16. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Ans:

For B^{-1}

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3-0) - 2(-1-0) - 2(2-0) = 3 + 2 - 4 = 1 \neq 0$$

i.e., B is invertible matrix

$\Rightarrow B^{-1}$ exist.

$A_{11} = 3, A_{12} = 1, A_{13} = 2$

$A_{21} = 2, A_{22} = 1, A_{23} = 2$

$A_{31} = 6, A_{32} = 2, A_{33} = 5$

$$adj(B) = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|B|}(adjb) = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now $(AB)^{-1} = B^{-1}.\, A^{-1}$

$$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

OBJECTIVE TYPE QUESTIONS (1 MARK)

1. If $\begin{vmatrix} 2x & -1 \\ 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}$, then the – value of x is

(a) 3 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{1}{4}$

2. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then the – value of x is

(a) 3 (b) ± 6 (c) ± 3 (d) 6

3. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the – value of x is

(a) 3 (b) –3 (c) ± 3 (d) 0

4. The value $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(a) –7 (b) 7 (c) 8 (d) 10

5. If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then the value of $\Delta + \Delta_1$ is

(a) 0 (b) –1 (c) 1 (d) none of these

6. The value of $\begin{vmatrix} \cos ec^2 x & \cot^2 x & 1 \\ \cot^2 x & \cos ec^2 x & -1 \\ 42 & 40 & 2 \end{vmatrix}$ is

(a) 0 (b) –1 (c) 1 (d) none of these

7. The value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ is

(a) 0 (b) –1 (c) 1 (d) none of these

8. If $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$ then

(a) $\Delta_1 = -\Delta$ (b) $\Delta \neq \Delta_1$ (c) $\Delta - \Delta_1 = 0$ (d) none of these

9. If $\Delta = \begin{vmatrix} Ax^2 & x^3 & 1 \\ By^2 & y^3 & 1 \\ Cz^2 & z^3 & 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ zy & zx & xy \end{vmatrix}$ then

(a) $\Delta_1 = -\Delta$ (b) $\Delta \neq \Delta_1$ (c) $\Delta - \Delta_1 = 0$ (d) $\Delta = x\Delta_1$

10. If x, y $\in$ R, then the determinant $\begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval

(a) $\sqrt{2}, \sqrt{2}$ (b) [–1, 1] (c) $\sqrt{2}$,1 (d) 1, $\sqrt{2}$

11. The determinant $\Delta = \begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$ is equal to

(a) 0 (b) –1 (c) 1 (d) none of these

12. The determinant $\Delta = \begin{vmatrix} \sqrt{37}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{74} & 5 & \sqrt{10} \\ 3+\sqrt{185} & \sqrt{15} & 5 \end{vmatrix}$ is equal to

(a) 0 (b) –1 (c) 1 (d) none of these

13. The determinant $\Delta = \begin{vmatrix} \sin^2 23^0 & \sin^2 67^0 & \cos 180^0 \\ -\sin^2 67^0 & -\sin^2 23^0 & \cos^2 180^0 \\ \cos 180^0 & \sin^2 23^0 & \sin^2 67^0 \end{vmatrix}$ is equal to

(a) 0 (b) –1 (c) 1 (d) none of these

14. If A = $\begin{vmatrix} 0 & 1 & 3 \\ 1 & 2 & x \\ 2 & 3 & 1 \end{vmatrix}$ and $A^{-1} = \begin{vmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ -\frac{1}{2} & 3 & -\frac{3}{2} \\ \frac{1}{2} & y & \frac{1}{2} \end{vmatrix}$, then the values of x and y are

(a) x = 0, y = 0 (b) x = 1, y = 1 (c) x = –1, y = 1 (d) x = 1, y = –1

15. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

(a) $a^3 + b^3 + c^3$ (b) 3 bc (c) $a^3 + b^3 + c^3 - 3abc$ (d) none of these

16. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be

(a) 9 (b) 3 (c) – 9 (d) 6

17. The determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ equals

(a) abc (b–c) (c – a) (a – b) (b) (b–c) (c – a) (a – b)

(c) (a + b + c) (b – c) (c – a) (a – b) (d) None of these

18. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) 3

19. If A, B and C are angles of a triangle, then the determinant $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to

(a) 0 (b) – 1 (c) 1 (d) None of these

20. If A, B and C are angles of a triangle, then the determinant $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$ is equal to

(a) 0 (b) – 1 (c) 1 (d) None of these

21. Let f (t) = $\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then $\lim\limits_{t \to 0} \frac{f(t)}{t^2}$ is equal to

(a) 0 (b) – 1 (c) 2 (d) 3

22. Let f (x) = $\begin{vmatrix} \cos x & 2\sin x & \sin x \\ x & x & x \\ 1 & 2x & x \end{vmatrix}$, then $\lim\limits_{x \to 0} \frac{f(x)}{x^2}$ is equal to

(a) 0 (b) – 1 (c) 2 (d) 3

23. The maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (θ is real number)

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\frac{2\sqrt{3}}{4}$

24. If f (x) = $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then

(a) f (a) = 0 (b) f (b) = 0 (c) f (0) = 0 (d) f (1) = 0

25. If A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if

(a) $\lambda = 2$ (b) $\lambda \neq 2$ (c) $\lambda \neq -2$ (d) None of these

26. If A and B are invertible matrices, then which of the following is not correct?

(a) adj A = |A|. A^{-1} (b) $\det(a)^{-1} = [\det(a)]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

27. If x, y, z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then value of $x^{-1} + y^{-1} + z^{-1}$ is

(a) xyz (b) $x^{-1} y^{-1} z^{-1}$ (c) – x – y – z (d) –1

28. The value of the determinant $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is

(a) $9x^2 (x + y)$ (b) $9y^2 (x + y)$ (c) $3y^2 (x + y)$ (d) $7x^2 (x + y)$

29. There are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then sum of these number is

(a) 4 (b) 5 (c) – 4 (d) 9

30. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactor of a_{ij}, then the value of Δ is given by

(a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (b) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

31. If A is a matrix of order 3 × 3, then |KA| = ______.
(a) 0 (b) –1 (c) 2 (d) 3

32. A and B are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If |A| = 2, then |B| is

(a) 16 (b) 4 (c) 6 (d) $\frac{1}{16}$

33. If a, b, c are all distinct, and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is

(a) 0 (b) –1 (c) 3 (d) –3

34. Let A be a square matrix of order 2 × 2, then |KA| is equal to
(a) K|A| (b) K^2|A| (c) K^3|A| (d) 2K|A|

35. Let A be a non-angular square matrix of order 3 × 3, then |A . adj A| is equal to
(a) $|A|^3$ (b) $|A|^2$ (c) |A| (d) 3|A|

36. Let A be a square matrix of order 3 × 3 and k a scalar, then | kA | is equal to
(a) k | A | (b) | k | | A | (c) k^3 | A | (d) none of these

37. A is invertible matrix of order 3 × 3 and |A| = 9, then value of $|A^{-1}|$ is _________.

38. If area of a triangle with vertices (3, 2), (–1, 4) and (6, k) is 7 sq units, then possible values of k are___.

39. If A and B are invertible matrices of the same order $(AB)^{-1}$ is __________.

40. If A is a matrix of order 3 × 3, then |3A| = _______ .

41. If A is invertible matrix of order 3 × 3, then $|A^{-1}|$ _______ .

42. If x, y, z ∈ R, then the value of determinant $\begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$ is equal to _______.

43. If cos2θ = 0, then $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$ = ________.

44. If A is a matrix of order 3 × 3, then $(A^2)^{-1}$ = ________.

45. If A is a matrix of order 3 × 3, then number of minors in determinant of A are ________.

46. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to _________.

47. If x = – 9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then other two roots are __________.

48. If A, B, C are the angles of a triangle, then $\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ = __________.

49. The value of $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix}$ is _________.

50. Maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix}$, where θ is a real number is ________.

CONTINUITY & DIFFERENTIABILITY

CHAPTER – 5: CONTINUITY AND DIFFERENTIABILITY

MARKS WEIGHTAGE – 08 marks

NCERT Important Questions & Answers

1. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}2x+3, & if \quad x\le 2\\ 2x-3, & if \quad x>2\end{cases}$.

Ans.

Here, $f(x)=\begin{cases}2x+3, & if \quad x\le 2\\ 2x-3, & if \quad x>2\end{cases}$.

At x =2, $LHL=\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-}(2x+3)$

Putting $x = 2 - h$ has $x \to 2^-$ when $h \to 0$

$\therefore \lim\limits_{x\to 2^-} f(x)=\lim\limits_{h\to 0}(2(2-h)+3)=\lim\limits_{h\to 0}(4-2h+3)=\lim\limits_{h\to 0}(7-2h)=7$

At x =2, $RHL=\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}(2x-3)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim\limits_{x\to 2^+} f(x)=\lim\limits_{h\to 0}(2(2+h)-3)=\lim\limits_{h\to 0}(4+2h-3)=\lim\limits_{h\to 0}(1+2h)=1$

$\therefore$ LHL $\ne$ RHL. Thus, $f(x)$ is discontinuous at $x = 2$.

2. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}\dfrac{|x|}{x}, & if \quad x\ne 0\\ 0, & if \quad x=0\end{cases}$

Ans. Here, $f(x)=\begin{cases}\dfrac{|x|}{x}, & if \quad x\ne 0\\ 0, & if \quad x=0\end{cases}$

$LHL=\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\dfrac{|x|}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim\limits_{x\to 0^-} f(x)=\lim\limits_{h\to 0}\dfrac{|0-h|}{0-h}=\lim\limits_{h\to 0}\dfrac{h}{-h}=-1$

$RHL=\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\dfrac{|x|}{x}$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim\limits_{x\to 0^+} f(x)=\lim\limits_{h\to 0}\dfrac{|0+h|}{0+h}=\lim\limits_{h\to 0}\dfrac{h}{h}=1$

$\therefore$ LHL $\ne$ RHL. Thus, $f(x)$ is discontinuous at $x = 0$.

3. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}x^3-3, & if \quad x\le 2\\ x^2+1, & if \quad x>2\end{cases}$

Ans.

For $x < 2$, $f(x) = x^3 - 3$ and for $x > 2$, $f(x) = x^2 + 1$ is a polynomial function, so f is continuous in the above interval. Therefore, we have to check the continuity at $x = 2$.

$LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 3)$

Putting $x = 2 - h$ has $x \to 2^-$ when $h \to 0$

$\therefore \lim_{x \to 2^-} f(x) = \lim_{h \to 0} ((2-h)^3 - 3) = \lim_{h \to 0} (8 - 12h + 6h^2 - h^3 - 3)$

$= \lim_{h \to 0} (5 - 12h + 6h^2 - h^3) = 5$

$RHL = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim_{x \to 2^+} f(x) = \lim_{h \to 0} ((2+h)^2 + 1) = \lim_{h \to 0} (4 + 4h + h^2 + 1)$

$= \lim_{h \to 0} (5 + 4h + h^2) = 5$

Also, $f(2) = (2)^3 - 3 = 8 - 3 = 5$ [since $f(x) = x^3 - 3$]

$\therefore$ LHL = RHL = f(2) . Thus, $f(x)$ is continuous at $x = 2$.

Hence, there is no point of discontinuity for this function $f(x)$.

4. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1, & if \quad x \le 3 \\ bx+3, & if \quad x > 3 \end{cases}$ **is continuous at $x = 3$.**

Ans.

Here, $f(x) = \begin{cases} ax+1, & if \quad x \le 3 \\ bx+3, & if \quad x > 3 \end{cases}$

$LHL = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax + 1)$

Putting $x = 3 - h$ has $x \to 3^-$ when $h \to 0$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{h \to 0} (a(3-h) + 1) = \lim_{h \to 0} (3a - ah + 1) = 3a + 1$

$RHL = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (bx + 3)$

Putting $x = 3 + h$ as $x \to 3^+$ when $h \to 0$

$\therefore \lim_{x \to 3^+} f(x) = \lim_{h \to 0} (b(3+h) + 3) = \lim_{h \to 0} (3b + bh + 3) = 3b + 3$

Also, f(3) =3 a +1 [since $f(x) = ax + 1$]

Since, $f(x)$ is continuous at $x = 3$.

$\therefore$ LHL = RHL = f(3)

$\Rightarrow 3a + 1 = 3b + 3 \Rightarrow 3a = 3b + 2 \Rightarrow a = b + \frac{2}{3}$

5. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x), & if \quad x \le 0 \\ 4x+1, & if \quad x > 0 \end{cases}$ **continuous at x = 0? What about continuity at x = 1?**

Ans.

Here, $f(x) = \begin{cases} \lambda(x^2 - 2x), & if \quad x \le 0 \\ 4x+1, & if \quad x > 0 \end{cases}$

At x = 0, $LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \lambda(x^2 - 2x)$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \lambda[(0-h)^2 - 2(0-h)] = \lim_{h \to 0} \lambda(h^2 + 2h) = 0$

At x = 0, $RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (4x+1)$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{h \to 0} [4(0+h)+1] = \lim_{h \to 0} (4h+1) = 1$

$\therefore$ LHL $\neq$ RHL. Thus, $f(x)$ is discontinuous at $x = 0$ for any value of λ.

At x = 1, $LHL = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (4x+1)$

Putting $x = 1 - h$ as $x \to 1^-$ when $h \to 0$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{h \to 0} (4(1+h)+1) = \lim_{h \to 0} (4+4h+1) = 5$

At x = 1, $RHL = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4x+1)$

Putting $x = 1 + h$ as $x \to 1^+$; $h \to 0$

$\therefore \lim_{x \to 1^+} f(x) = \lim_{h \to 0} [4(1+h)+1] = \lim_{h \to 0} (4+4h+1) = 5$

$\therefore$ LHL = RHL. Thus, $f(x)$ is continuous at $x = 1$ for any value of λ.

6. Find all points of discontinuity of *f*, where $f(x) = \begin{cases} \dfrac{\sin x}{x}, & if \quad x < 0 \\ x+1, & if \quad x \geq 0 \end{cases}$

Ans. Here, $f(x) = \begin{cases} \dfrac{\sin x}{x}, & if \quad x < 0 \\ x+1, & if \quad x \geq 0 \end{cases}$

At x = 0, $LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \dfrac{\sin x}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \dfrac{\sin(0-h)}{0-h} = \lim_{h \to 0} \dfrac{\sin(-h)}{-h} = \lim_{h \to 0} \dfrac{-\sin h}{-h} = \lim_{h \to 0} \dfrac{\sin h}{h} = 1$

At x = 0, $RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \dfrac{\sin x}{x}$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{h \to 0} \dfrac{\sin(0+h)}{0+h} = \lim_{h \to 0} \dfrac{\sin h}{h} = 1$

Also, f(0) = 0 + 1 = 1

$\therefore$ LHL = RHL = f(0). Thus, $f(x)$ is continuous at $x = 0$.

When $x < 0$, sin x and x both are continuous. Therefore, $\dfrac{\sin x}{x}$ is also continuous.

When $x > 0$, $f(x) = x + 1$ is a polynomial. Therefore f is continuous.

Hence, there is no point of discontinuity for this function $f(x)$.

7. **Determine if *f* defined by** $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if \quad x\neq 0 \\ 0, & if \quad x=0 \end{cases}$ **is a continuous function?**

Ans.

Here, $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if \quad x\neq 0 \\ 0, & if \quad x=0 \end{cases}$

At x = 0, $LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 \sin\frac{1}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{h\to 0}(0-h)^2 \sin\frac{1}{0-h} = \lim_{h\to 0}\left(-h^2\sin\frac{1}{h}\right) = -0\times\sin\infty$

= 0 x value between –1 and 1 (since $-1 \le \sin x \le 1$, for all values of $x \in R$)

At x = 0, $RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 \sin\frac{1}{x}$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x\to 0^+} f(x) = \lim_{h\to 0}(0+h)^2 \sin\frac{1}{0+h} = \lim_{h\to 0}\left(h^2\sin\frac{1}{h}\right) = 0\times\sin\infty$

= 0 x value between –1 and 1 (since $-1 \le \sin x \le 1$, for all values of $x \in R$)

$\therefore$ LHL = RHL = f(0). Thus, $f(x)$ is continuous at $x = 0$.

8. **Find the values of *k* so that the function *f*** $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if \quad x\neq \frac{\pi}{2} \\ 3, & if \quad x=\frac{\pi}{2} \end{cases}$ **is continuous at point** $x=\frac{\pi}{2}$

Ans.

Here, $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if \quad x\neq \frac{\pi}{2} \\ 3, & if \quad x=\frac{\pi}{2} \end{cases}$

$LHL = \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^-} \frac{k\cos x}{\pi-2x}$

Putting $x = \frac{\pi}{2} - h$ as $x \to \frac{\pi}{2}^-$ when $h \to 0$

$\therefore \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to 0}\frac{ksinh}{2h} = \frac{k}{2}\times\lim_{h\to 0}\frac{sinh}{h} = \frac{k}{2}\times 1 = \frac{k}{2}$

Since f(x) is continuous at $x=\frac{\pi}{2}$, therefore LHL = $f\left(\frac{\pi}{2}\right)$

Also, $f\left(\frac{\pi}{2}\right) = 3 \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$

9. Find the values of *k* so that the function *f* $f(x)=\begin{cases} kx+1, & if \quad x \le 5 \\ 3x-5, & if \quad x>5 \end{cases}$ is continuous at point x = 5.

Ans.

Here, $f(x)=\begin{cases} kx+1, & if \quad x \le 5 \\ 3x-5, & if \quad x>5 \end{cases}$

At x = 5, $LHL = \lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (kx+1)$

Putting $x = 5 - h$ has $x \to 5^-$ when $h \to 0$

$\therefore \lim_{x \to 5^-} f(x) = \lim_{h \to 0} (k(5-h)+1) = \lim_{h \to 0} (5k-kh+1) = 5k+1$

At x = 5, $RHL = \lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (3x-5)$

Putting $x = 5 + h$ as $x \to 5^+$; $h \to 0$

$\therefore \lim_{x \to 5^+} f(x) = \lim_{h \to 0} (3(5+h)-5) = \lim_{h \to 0} (10+3h) = 10$

Also, f(5) = 5k + 1

Since f(x) is continuous at x = 5, therefore LHL = RHL = f(5)

$\Rightarrow 5k+1=10 \Rightarrow 5k=9 \Rightarrow k=\frac{9}{5}$

10. Find the values of *a* and *b* such that the function defined by $f(x)=\begin{cases} 5, & if \quad x \le 2 \\ ax+b, & if \quad 2<x<10 \\ 21, & if \quad x \ge 10 \end{cases}$ is a continuous function.

Ans.

Here, $f(x)=\begin{cases} 5, & if \quad x \le 2 \\ ax+b, & if \quad 2<x<10 \\ 21, & if \quad x \ge 10 \end{cases}$

At x =2, $LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (5) = 5$

At x =2, $RHL = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (ax+b)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim_{x \to 2^+} f(x) = \lim_{h \to 0} (a(2+h)+b) = \lim_{h \to 0} (2a+ah+b) = 2a+b$

Also, f(2) = 5

Since $f(x)$ is continuous at x = 2, therefore LHL = RHL = f(2)

$\Rightarrow 2a+b=5$ ----------------- (1)

At x =10, $LHL = \lim_{x \to 10^-} f(x) = \lim_{x \to 10^-} (ax+b)$

Putting $x = 10 - h$ has $x \to 10^-$ when $h \to 0$

$\therefore \lim_{x \to 10^-} f(x) = \lim_{h \to 0} (a(10-h)+b) = \lim_{h \to 0} (10a-ah+b) = 10a+b$

At x =2, $RHL = \lim_{x \to 10^+} f(x) = \lim_{x \to 10^+} (21) = 21$

Also, f(10) = 21

Since $f(x)$ is continuous at x = 10, therefore LHL = RHL = f(10)

Since, $f(x)$ is continuous at x = 10.

LHL = RHL = f(10)

$\Rightarrow$ 10a + b = 21 ………………(2)

Subtracting Eq. (1) from Eq. (2), we get $8a = 16 \Rightarrow a = 2$

Put a =2 in Eq. (1), we get $2 \times 2 + b = 5 \Rightarrow b = 1$

11. Prove that the function *f* given by *f* (*x*) = | *x* – 1 |, *x* ∈ R is not differentiable at *x* = 1.

Ans.

Given, $f(x) = |x-1| = \begin{cases} x-1, & if \quad x-1 \ge 0 \\ -(x-1), & if \quad x-1 < 0 \end{cases}$

We have to check the differentiability at x = 1

Here, $f(1) = 1 - 1 = 0$

$$Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{1-(1-h)-0)}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h} = -1$$

and

$$Rf'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)-1-0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

$\therefore Lf'(1) \ne Rf'(1)$.

Hence, f(x) is not differentiable at x = 1

12. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans.

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}\cos 2\theta = 2\theta$$

$$\Rightarrow y = 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

13. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans.

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \sin^{-1}\cos 2\theta = \sin^{-1}\sin\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow y = \frac{\pi}{2} - 2\theta \Rightarrow y = \frac{\pi}{2} - 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$$

14. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

Ans.

Let x = $\tan\theta \;\Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}\sin 2\theta = \cos^{-1}\cos\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow y = \frac{\pi}{2} - 2\theta \Rightarrow y = \frac{\pi}{2} - 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$$

15. Find $\frac{dy}{dx}$ if $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Ans.

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$, then we have

$$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}\sec 2\theta = 2\theta$$

$$\Rightarrow y = 2\cos^{-1} x = 2 \times \frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

16. Differentiate sin ($\tan^{-1} e^{-x}$) with respect to x.

Ans.

Let y = sin ($\tan^{-1} e^{-x}$)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(\tan^{-1} e^{-x})\right] = \cos(\tan^{-1} e^{-x})\frac{d}{dx}(\tan^{-1} e^{-x})$$

$$= \cos(\tan^{-1} e^{-x})\frac{1}{1+(e^{-x})^2}\frac{d}{dx}(e^{-x})$$

$$= \cos(\tan^{-1} e^{-x})\frac{1}{1+e^{-2x}}(-e^{-x}) = -\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$$

17. Differentiate log (cos e^x) with respect to x.

Ans.

Let y = log (cos e^x)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\cos e^x)\right] = \frac{1}{\cos e^x}\frac{d}{dx}(\cos e^x)$$

$$= \frac{1}{\cos e^x}(-\sin e^x)\frac{d}{dx}(e^x) = (-\tan e^x).e^x = -e^x \tan e^x$$

18. Differentiate cos (log x + e^x), $x > 0$ with respect to x.

Ans.

Let y = cos (log x + e^x)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\cos(\log x + e^x)\right] = -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x)$$

$$= -\sin(\log x + e^x)\left(\frac{1}{x} + e^x\right) = -\sin(\log x + e^x)\left(\frac{1 + xe^x}{x}\right)$$

$$= \frac{-(1 + xe^x)\sin(\log x + e^x)}{x}$$

19. Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$.

Ans.

Given that $y^x + x^y + x^x = a^b$

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

Therefore, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ------------------ (1)

Now, $u = y^x$. Taking logarithm on both sides, we have log $u = x$ log y

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x) = x\frac{1}{y}.\frac{dy}{dx} + \log y.1$$

$$\Rightarrow \frac{du}{dx} = u\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \text{ -------------------- (2)}$$

Also $v = x^y$

Taking logarithm on both sides, we have log $v = y$ log x

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx} = y\frac{1}{x} + \log x\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = v\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) \text{ ------------------ (3)}$$

Again $w = x^x$

Taking logarithm on both sides, we have log $w = x$ log x.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{w}\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) = x\frac{1}{x} + \log x.1$$

$$\Rightarrow \frac{dw}{dx} = w(1 + \log x) = x^x(1 + \log x) \text{ -------------------- (4)}$$

From (1), (2), (3), (4), we have

$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$

$$(x.y^{x-1} + x^y.\log x)\frac{dy}{dx} = -x^x(1 + \log x) - y.x^{y-1} - y^x\log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left[x^x(1 + \log x) + y.x^{y-1} + y^x\log y\right]}{x.y^{x-1} + x^y.\log x}$$

20. Differentiate $x^x - 2^{\sin x}$ with respect to x.

Ans.

Let y = $x^x - 2^{\sin x}$

Let u = x^x and v = $2^{\sin x}$ then we have y = u – v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ -------------------- (1)

Now, $u = x^x$

Taking logarithm on both sides, we have log u = x log x.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) = x\frac{1}{x} + \log x.1$$

$$\Rightarrow \frac{du}{dx} = u\left(1+\log x\right) = x^x\left(1+\log x\right) \quad \text{--------------------} \quad (2)$$

Again $v = 2^{sinx}$

Taking logarithm on both sides, we have log v = (sin$x$) log *2*.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = \cos x(\log 2) \Rightarrow \frac{dv}{dx} = v[\cos x(\log 2)] = 2^{\sin x}[\cos x(\log 2)]$$

From (1), (2) and (3), we get

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} = x^x\left(1+\log x\right) - 2^{\sin x}[\cos x(\log 2)]$$

21. Differentiate (log x)^x + x$^{\log x}$ with respect to x.

Ans.

Let y = (log x)^x + x$^{\log x}$

Let u = (log x)^x and v = x$^{\log x}$ then we have y = u + v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, u = (logx)^x

Taking logarithm on both sides, we have log u = xlog(logx).

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x) = \frac{x}{\log x}\times\frac{1}{x} + \log(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right] = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \quad \text{----------------------} \ (2)$$

Again v = x^{logx}

Taking logarithm on both sides, we have log v = (log$x$) logx = (logx)^2

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = 2\log x\frac{d}{dx}(\log x) = 2\log x\times\frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{2\log x}{x}\right] = x^{\log x}\left[\frac{2\log x}{x}\right] \quad \text{----------------------} \ (3)$$

From (1), (2) and (3)

$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{x-1}\left[1+\log x\log(\log x)\right] + 2x^{\log x-1}.\log x$$

22. Differentiate $(\sin x)^x + \sin^{-1}\sqrt{x}$ **with respect to x.**

Ans.

Let y = $(\sin x)^x + \sin^{-1}\sqrt{x}$

Let u = $(\sin x)^x$, v = $\sin^{-1}\sqrt{x}$ then we have y = u + v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, u = (sin x)^x

Taking logarithm on both sides, we have log u = xlog(sin x).

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\sin x) + \log(\sin x)\frac{d}{dx}(x) = \frac{x}{\sin x}\times\cos x + \log(\sin x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x\cot x + \log(\sin x)\right] = (\sin x)^x\left[x\cot x + \log(\sin x)\right] \quad \text{------------- (2)}$$

Again $v = \sin^{-1}\sqrt{x}$

Differentiating both sides w.r.t. x, we have

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}(\sqrt{x}) = \frac{1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}} \quad \text{---------------- (3)}$$

From (1), (2) and (3)

$$\frac{dy}{dx} = (\sin x)^x\left[x\cot x + \log(\sin x)\right] + \frac{1}{2\sqrt{x-x^2}}$$

23. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

Ans.

Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Let $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$ then we have $y = u + v$

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, $u = x^{sinx}$

Taking logarithm on both sides, we have log u = (sinx) logx.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = \sin x\frac{d}{dx}\log x + \log x\frac{d}{dx}(\sin x) = \sin x\frac{1}{x} + \log x\cos x$$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{\sin x}{x} + \log x\cos x\right] = x^{\sin x}\left[\frac{\sin x}{x} + \log x\cos x\right] \quad \text{---------------------- (2)}$$

Again $v = \sin x^{\cos x}$

Taking logarithm on both sides, we have log v = (cos$x$) log(sinx)

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = cosx\frac{d}{dx}\log(\sin x) + \log(\sin x)\frac{d}{dx}(\cos x) = \cos x\frac{1}{\sin x}\times\cos x + \log(\sin x)(-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v\left[\cot x\cos x - \sin x\log(\sin x)\right] = \sin x^{\cos x}\left[\cot x\cos x - \sin x\log(\sin x)\right] \quad \text{----------- (3)}$$

From (1), (2) and (3)

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x\cos x\right] + \sin x^{\cos x}\left[\cot x\cos x - \sin x\log(\sin x)\right]$$

24. Find $\frac{dy}{dx}$ if $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

Ans.

Given that $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = a(1+\cos\theta), \frac{dy}{d\theta} = a(\sin\theta)$$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(\sin\theta)}{a(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$

25. Find $\frac{dy}{dx}$ if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

Ans.

Given that $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = -\sin\theta - (-\sin 2\theta)\times 2 = -\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{d\theta} = \cos\theta - (\cos 2\theta)\times 2 = \cos\theta - 2\cos 2\theta$$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta + 2\sin 2\theta}{\cos\theta - 2\cos 2\theta}$

26. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Ans.

Given that $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = a(1-\cos\theta), \frac{dy}{d\theta} = a(0-\sin\theta) = -a\sin\theta$$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{1-\cos\theta} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = -\cot\frac{\theta}{2}$

27. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

Ans.

Given that $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$

Multiplying both we get,

$$xy = \sqrt{a^{\sin^{-1}t}}\sqrt{a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t}.a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t+\cos^{-1}t}} = \sqrt{a^{\pi/2}}$$

Differentiating both sides w.r.t. x, we get

$$x\frac{dy}{dx} + y = 0 \Rightarrow x\frac{dy}{dx} = -y \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

28. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Ans.

Given that $y = 3e^{2x} + 2e^{3x}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 6e^{2x} + 6^{3x} = 6(e^{2x} + e^{3x})$$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 6(2e^{2x} + 3e^{3x})$$

Now, $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6(2e^{2x} + 3e^{3x}) - 5(6(e^{2x} + e^{3x})) + 6(3e^{2x} + 2e^{3x})$

$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x} = 0$

29. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$

Ans.

Given that $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = y_1 = -3\sin(\log x)\frac{d}{dx}(\log x) + 4\cos(\log x)\frac{d}{dx}(\log x)$$

$$= -3\sin(\log x)\frac{1}{x} + 4\cos(\log x)\frac{1}{x} = \frac{1}{x}\left[-3\sin(\log x) + 4\cos(\log x)\right]$$

$$\Rightarrow xy_1 = -3\sin(\log x) + 4\cos(\log x)$$

Again, Differentiating both sides w.r.t. x, we get

$$xy_2 + y_1.1 = -3\cos(\log x)\frac{d}{dx}(\log x) - 4\sin(\log x)\frac{d}{dx}(\log x)$$

$$= -3\cos(\log x)\frac{1}{x} - 4\sin(\log x)\frac{1}{x} = -\frac{1}{x}\left[3\cos(\log x) + 4\sin(\log x)\right] = -\frac{y}{x}$$

$$\Rightarrow x^2 y_2 + xy_1 = -y$$

$$\Rightarrow x^2 y_2 + xy_1 + y = 0$$

30. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Ans.

Given that $e^y(x+1) = 1$

$$\Rightarrow e^y = \frac{1}{x+1}$$

Differentiating both sides w.r.t. x, we get

$$e^y\frac{dy}{dx} = -\frac{1}{(x+1)^2} \Rightarrow \frac{1}{x+1}\frac{dy}{dx} = -\frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(-\frac{1}{x+1}\right)^2 = \left(\frac{dy}{dx}\right)^2$$

31. If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2$

Ans.

Given that $y = (\tan^{-1}x)^2$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2\tan^{-1}x\frac{d}{dx}(\tan^{-1}x) = 2\tan^{-1}x \times \frac{1}{1+x^2}$$

$$\Rightarrow y_1 = \frac{2\tan^{-1}x}{1+x^2} \Rightarrow (1+x^2)y_1 = 2\tan^{-1}x$$

Again, Differentiating both sides w.r.t. x, we get

$$(1+x^2)y_2 + 2xy_1 = 2\times\frac{1}{1+x^2} \Rightarrow (1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

32. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for , – 1 < x < 1, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

Ans.

Given that $x\sqrt{1+y} + y\sqrt{1+x} = 0 \;\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides, we get

$$x^2(1+y) = y^2(1+x) \qquad \Rightarrow x^2 - y^2 + x^2y - y^2x = 0$$

$$\Rightarrow (x-y)(x+y)+xy(x-y)=0 \quad \Rightarrow (x-y)(x+y+xy)=0$$

$$\Rightarrow x-y=0 \;\; or \;\; x+y+xy=0$$

$$\Rightarrow y=x \;\; or \;\; y(1+x)=-x \Rightarrow y=x \;\; or \;\; y=\frac{-x}{1+x}$$

But y =x does not satisfy the given equation

So, we consider, $y=\dfrac{-x}{1+x}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{-x}{x+1}\right)=-\frac{(1+x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+x)}{(1+x)^2}=-\frac{(1+x)-x}{(1+x)^2}=-\frac{1}{(1+x)^2}$$

33. If cos *y* = *x* cos (*a* + *y*), with cos *a* ≠ ± 1, prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$

Ans.

Given that cos y = x cos (a + y)$\Rightarrow x=\dfrac{\cos y}{\cos(a+y)}$

Differentiating both sides w.r.t. y, we get

$$\frac{dx}{dy}=\frac{d}{dy}\left(\frac{\cos y}{\cos(a+y)}\right)=\frac{\cos(a+y)(-\sin y)-\cos y[-\sin(a+y)]}{\cos^2(a+y)}$$

$$=\frac{\sin(a+y)\cos y-\cos(a+y)\sin y}{\cos^2(a+y)}=\frac{\sin(a+y-y)}{\cos^2(a+y)}=\frac{\sin a}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$

34. If *x* = *a* (cos *t* + *t* sin *t*) and *y* = *a* (sin *t* – *t* cos *t*), find $\dfrac{d^2y}{dx^2}$

Ans.

Given that x = a (cos t + t sin t) and y = a (sin t – t cos t)

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt}=a\left[\frac{d}{dt}\cos t+\left(t\frac{d}{dt}\sin t+\sin t\frac{d}{dt}(t)\right)\right]=a[-\sin t+(t\sin t+\sin t)]=at\cos t$$

and $\dfrac{dy}{dt}=a\left[\dfrac{d}{dt}\sin t+\left(t\dfrac{d}{dt}\cos t+\cos t\dfrac{d}{dt}(t)\right)\right]=a[\cos t-(-t\sin t+\cos t)]=at\sin t$

Now, $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{at\sin t}{at\cos t}=\tan t$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\tan t=\sec^2 t\times\frac{dt}{dx}=\frac{\sec^2 t}{at\cos t}=\frac{1}{at}\sec^3 t$$

35. Differentiate $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ w.r.t. x

Ans.

Let y $=\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)=\tan^{-1}\left(\dfrac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}(\tan\frac{x}{2}) = \frac{x}{2}$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

36. Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ **w.r.t. x**

Ans.

Let y = $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x \times 2}{1+(2^x)^2}\right)$

Let $2^x = \tan\theta \Rightarrow \theta = \tan^{-1} 2^x$ then we have

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} 2^x$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2 \times \frac{1}{1+(2^x)^2}\frac{d}{dx}(2^x) = \frac{2}{1+4^x} 2^x \log 2 = \frac{2^{x+1}\log 2}{1+4^x}$$

37. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

Ans.

Let u = $\sin^2 x$ and v = $e^{\cos x}$

Differentiating u and v w.r.t. x, we get

$$\frac{du}{dx} = 2\sin x \frac{d}{dx}(\sin x) = 2\sin x \cos x$$

and $\frac{dv}{dx} = e^{\cos x}\frac{d}{dx}(\cos x) = e^{\cos x}(-\sin x) = (-\sin x)e^{\cos x}$

Now, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x\cos x}{(-\sin x)e^{\cos x}} = -\frac{2\cos x}{e^{\cos x}}$

CHAPTER – 5: CONTINUITY AND DIFFERENTIABILITY

MARKS WEIGHTAGE – 08 marks

Previous Years Board Exam (Important Questions)

1. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x+1 \ ; & x<2 \\ k \ ; & x=2 \\ 3x-1 \ ; & x>2 \end{cases}$$

2. Discuss the continuity of the following function at $x = 0$: $f(x) = \begin{cases} \dfrac{x^4+2x^3+x^2}{\tan^{-1}x}, & x \neq 0 \\ 0 & , x=0 \end{cases}$

3. If the function $f(x)$ given by $f(x) = \begin{cases} 3ax+b \ ; & x>1 \\ 11 \ ; & x=1 \\ 5ax-2b \ ; & x<1 \end{cases}$ is continuous at $x = 1$, find the values of a and b.

4. Find the relationship between 'a' and 'b' so that the function 'f' defined by:

$f(x) = \begin{cases} ax+1, & x \leq 3 \\ bx+3, & x>3 \end{cases}$ is continuous at $x = 3$.

5. Show that the function $f(x) = |x-3|$, $x \in R$, is continuous but not differentiable at $x = 3$.

6. Find the value of k, for which $f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \dfrac{2x+1}{x-1}, & 0 \leq x < 1 \end{cases}$ is continuous at $x = 0$.

7. Find the value of k so that the function f, defined by $f(x) = \begin{cases} kx+1, & x \leq \pi \\ \cos x, & x>\pi \end{cases}$ is continuous at $x = \pi$.

8. Find the value of 'a' for which the function f defined as

$f(x) = \begin{cases} a\sin\dfrac{\pi}{2}(x+1), & x \leq 0 \\ \dfrac{\tan x - \sin x}{x^3}, & x>0 \end{cases}$ is continuous at $x = 0$.

9. Find all points of discontinuity of f, where f is defined as follows :

$$f(x) = \begin{cases} |x|+3 \ ; & x \leq -3 \\ -2x \ ; & -3<x<3 \\ 6x+2 \ ; & x \geq 3 \end{cases}$$

10. Show that the function f defined as follows, is continuous at $x = 2$, but not differentiable:

$$f(x) = \begin{cases} 3x-2 \ ; & 0<x \leq 1 \\ 2x^2 - x; & 1<x \leq 2 \\ 5x-4 \ ; & x>2 \end{cases}$$

11. If $f(x) = \sqrt{\dfrac{\sec x-1}{\sec x+1}}$, then f'(x). Also find $f'\left(\dfrac{\pi}{2}\right)$.

12. Find $\dfrac{dy}{dx}$,if $(x^2+y^2)^2 = xy$.

13. If $\sin y = x\sin(a+y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$

14. If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.

15. If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$, then prove that $(1-x^2)\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} - y = 0$.

16. If $y = e^x(\sin x + \cos x)$, then show that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$.

17. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then find $\dfrac{d^2y}{dx^2}$.

18. If $\log(x^2+y^2) = 2\tan^{-1}\left(\dfrac{y}{x}\right)$, then show that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

19. If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

20. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

21. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

22. Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$

23. Differentiate the following with respect to x : $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

24. If $y = \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, find $\dfrac{dy}{dx}$

25. Differentiate the following function w.r.t. x : $(x)^{\cos x} + (\sin x)^{\tan x}$

26. If $y = e^{a\sin^{-1}x}, -1 \le x \le 1$, then show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$

27. If $y = \cos^{-1}\left(\dfrac{3x+4\sqrt{1-x^2}}{5}\right)$, find $\dfrac{dy}{dx}$

28. If $y = \operatorname{cosec}^{-1} x$, $x > 1$. then show that $x(x^2-1)\dfrac{d^2y}{dx^2} + (2x^2-1)\dfrac{dy}{dx} = 0$.

29. If $y = \log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$ then show that $\dfrac{dy}{dx} = \sec x$. Also find $\dfrac{d^2y}{dx^2}$ at $x = \dfrac{\pi}{4}$.

30. Differentiate the following function with respect to x: $f(x) = \tan^{-1}\left(\dfrac{1-x}{1+x}\right) - \tan^{-1}\left(\dfrac{x+2}{1-2x}\right)$

31. If $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right), y = a(1+\sin t)$, then find $\dfrac{d^2y}{dx^2}$.

32. If $x = a(\theta - \sin\theta), y = a(1+\cos\theta)$, then find $\dfrac{d^2y}{dx^2}$.

33. Differentiate $x^{\cos x} + \dfrac{x^2+1}{x^2-1}$ w.r.t.x

34. If $x^y = e^{x-y}$, then show that $\dfrac{dy}{dx} = \dfrac{\log x}{[\log(xe)]^2}$

35. If $x=\tan\left(\frac{1}{a}\log y\right)$, then show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0$.

36. Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$

37. If $y=\log\left[x+\sqrt{x^2+1}\right]$ then show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

38. If $y=\log\left[x+\sqrt{x^2+a^2}\right]$ then show that $(x^2+a^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

39. If $y=\sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$.

40. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ w.r.t. x.

41. If $x = a$ (cos t + t sin t) and $y = a$ (sin t – t cos t), $0<t<\frac{\pi}{2}$, find $\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2y}{dx^2}$.

42. If $x^m y^m=(x+y)^{m+n}$, then show that $\frac{dy}{dx}=\frac{y}{x}$.

43. If $x^{16}y^9=(x^2+y)^{17}$, then show that $\frac{dy}{dx}=\frac{2y}{x}$.

44. If $x=a\sin t, y=a\left(\cos t+\log\tan\frac{t}{2}\right)$, then find $\frac{d^2y}{dx^2}$.

45. If $y^x=e^{y-x}$, then show that $\frac{dy}{dx}=\frac{(1+\log y)^2}{\log y}$.

46. If $x^y=e^{x-y}$, then show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

47. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

48. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{6}$.

49. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.

50. If $y = \sin(\log x)$, then prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

51. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

52. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

53. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

54. If $y=x^x$, then show that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$

OBJECTIVE TYPE QUESTIONS (1 MARK)

1. The function f (x) = $\begin{cases} \dfrac{\sin x}{x} + \cos x, & if \quad x \neq 0 \\ k, & if \quad x = 0 \end{cases}$ is continuous at x = 0, then the value of k is

(a) 3 (b) 2 (c) 1 (d) 1.5

2. The function f (x) = [x], where [x] denotes the greatest integer function, is continuous at
(a) 4 (b) – 2 (c) 1 (d) 1.5

3. The number of points at which the function f (x) = $\dfrac{1}{x-[x]}$ is not continuous is

(a) 1 (b) 2 (c) 3 (d) none of these

4. The function given by f (x) = tanx is discontinuous on the set

(a) $\{n\pi : n \in Z\}$ (b) $\{2n\pi : n \in Z\}$ (c) $\left\{(2n+1)\dfrac{\pi}{2} : n \in Z\right\}$ (d) $\left\{\dfrac{n\pi}{2} : n \in Z\right\}$

5. Let f (x)= |cosx|. Then,
(a) f is everywhere differentiable.
(b) f is everywhere continuous but not differentiable at x = nπ, n ∈Z .
(c) f is everywhere continuous but not differentiable at x = (2n + 1)$\dfrac{\pi}{2}$
(d) none of these.

6. The function f (x) = |x| + |x – 1| is
(a) continuous at x = 0 as well as at x = 1. (b) continuous at x = 1 but not at x = 0.
(c) discontinuous at x = 0 as well as at x = 1. (d) continuous at x = 0 but not at x = 1.

7. The value of k which makes the function defined by f(x) = $\begin{cases} \sin\dfrac{1}{x}, & if \quad x \neq 0 \\ k, & if \quad x = 0 \end{cases}$, continuous at x = 0 is

(a) 8 (b) 1 (c) –1 (d) none of these

8. The set of points where the functions f given by f (x) = |x – 3| cosx is differentiable is
(a) R (b) R – {3} (c) (0, ∞) (d) none of these

9. Differential coefficient of sec ($\tan^{-1}$x) w.r.t. x is

(a) $\dfrac{x}{\sqrt{1+x^2}}$ (b) $\dfrac{x}{1+x^2}$ (c) $x\sqrt{1+x^2}$ (d) $\dfrac{1}{\sqrt{1+x^2}}$

10. If u = $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ and v = $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, then $\dfrac{du}{dv}$ is

(a) $\dfrac{1}{2}$ (b) x (c) $\dfrac{1-x^2}{1+x^2}$ (d) 1

11. A function f is said to be continuous for x ∈ R, if
(a) it is continuous at x = 0 (b) differentiable at x = 0
(c) continuous at two points (d) differentiable for x R

12. If f (x) = $\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, x ≠ 2 and f(x) = k for x = 2, then value of k for which f is continuous is

(a) –2 (b) –1 (c) 0 (d) 1

13. If f (x) = 2x and g (x) = $\frac{x^2}{2}+1$, then which of the following can be a discontinuous function

(a) f (x) + g (x) (b) f (x) – g (x) (c) f (x) . g (x) (d) $\frac{g(x)}{f(x)}$

14. The function f (x) = $\frac{1-x^2}{4x-x^3}$ is

(a) discontinuous at only one point (b) discontinuous at exactly two points
(c) discontinuous at exactly three points (d) none of these

15. The set of points where the function f given by f (x) = |2x-1| sinx is differentiable is

(a) R (b) R – $\left\{\frac{1}{2}\right\}$ (c) (0, ∞) (d) none of these

16. The function f (x) = cot x is discontinuous on the set

(a) $\{n\pi : n \in Z\}$ (b) $\{2n\pi : n \in Z\}$ (c) $\left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$ (d) $\left\{\frac{n\pi}{2} : n \in Z\right\}$

17. Let f (x)= | sinx |. Then,
(a) f is everywhere differentiable.
(b) f is everywhere continuous but not differentiable at x = nπ, n ∈Z .
(c) f is everywhere continuous but not differentiable at x = (2n + 1)$\frac{\pi}{2}$, n ∈Z
(d) none of these.

18. The function f (x) = $e^{|x|}$ is
(a) continuous everywhere but not differentiable at x = 0
(b) continuous and differentiable everywhere
(c) not continuous at x = 0
(d) none of these.

19. If f (x) = $x^2 \sin\frac{1}{x}$, where x ≠ 0, then the value of the function f at x = 0, so that the function is continuous at x = 0, is

(a) 0 (b) – 1 (c) 1 (d) none of these

20. If f(x) = $\begin{cases} mx+1, & if \quad x \le \frac{\pi}{2} \\ \sin x + n, & if \quad x > \frac{\pi}{2} \end{cases}$, is continuous at x = $\frac{\pi}{2}$, then

(a) m = 1, n = 0 (b) m = $\frac{n\pi}{2}+1$ (c) n = $\frac{m\pi}{2}$ (d) m = n = $\frac{\pi}{2}$

21. If y = $\log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to ______.

(a) $\frac{4x^3}{1-x^4}$ (b) $\frac{-4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $\frac{-4x^3}{1-x^4}$

22. If y = $\sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to ______.

(a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{2y-1}$ (d) $\frac{\sin x}{1-2y}$

23. The derivative of $\cos^{-1}(2x^2-1)$ w.r.t. $\cos^{-1}x$ is

(a) 2 (b) $\frac{-1}{2\sqrt{1-x^2}}$ (c) $\frac{2}{x}$ (d) $1-x^2$.

24. If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4}$

25. If the function $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & if \quad x \neq \frac{\pi}{2} \\ 3 \quad , & if \quad x = \frac{\pi}{2} \end{cases}$ be continuous at x = $\frac{\pi}{2}$, then k =

(a) 3 (b) 6 (c) 12 (d) none of these

26. In order that the function $f(x) = (x + 1)^{1/x}$ is continuous at x = 0, f(0) must be defined as

(a) f(0) = 0 (b) f(0) = e (c) f(0) = 1/e (d) f(0) = 1

27. Let $f(x) = \begin{cases} x^2 + k, & if \quad x \geq 0 \\ -x^2 - k, & if \quad x < 0 \end{cases}$. If the function f(x) be continuous at x = 0, then k =

(a) 0 (b) 1 (c) 2 (d) -2

28. The points at which the function $f(x) = \frac{x+1}{x^2+x-12}$ is discontinuous are

(a) -3, 4 (b) 3, -4 (c) -1, -3, 4 (d) -1, 3, 4

29. If f(x) = | x - b|, then function

(a) Is continuous at x = 1 (b) Is continuous at x = b
(c) Is discontinuous at x = b (d) None of these

30. If $f(x) = \begin{cases} 1+x, & if \quad x \leq 2 \\ 5-x, & if \quad x > 2 \end{cases}$, then

(a) f(x) is continuous at x = 2 (b) f(x) is discontinuous at x = 2
(c) f(x) is continuous at x = 3 (d) None of these

31. If function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & if \quad x \neq 1 \\ k, & if \quad x = 1 \end{cases}$ is continuous at x = 1, then the value of k will be

(a) -1 (b) 2 (c) -3 (d) -2

32. For the function $f(x)=\begin{cases}\dfrac{\sin^2 ax}{x^2}, & if \quad x\neq 0\\ 1, & if \quad x=0\end{cases}$ which one is a true statement

(a) f(x) is continuous at x = 0 (b) f(x) is discontinuous at x = 0, when a $\neq \pm 1$
(c) f(x) is continuous at x = a (d) None of these

33. If $f(x)=\begin{cases}\dfrac{\sin 2x}{5x}, & if \quad x\neq 0\\ k, & if \quad x=0\end{cases}$ is continuous at x = 0, then the value of k will be

(a) 1 (b) $\dfrac{2}{5}$ (c) $-\dfrac{2}{5}$ (d) None of these

34. If $f(x)=\begin{cases}\dfrac{|x-a|}{x-a}, & if \quad x\neq a\\ 1, & if \quad x=a\end{cases}$, then

(a) f(x) is continuous at x = a (b) f(x) is discontinuous at x = a
(c) $\lim\limits_{x\to a} f(x)=1$ (d) None of these

35. If $f(x)=\begin{cases}\dfrac{x^4-16}{x-2}, & if \quad x\neq 2\\ 16, & if \quad x=2\end{cases}$, then

(a) f(x) is continuous at x = 2 (b) f(x) is discontinuous at x = 2
(c) $\lim\limits_{x\to 2} f(x)=16$ (d) None of these

36. The number of points at which the function f (x) = $\dfrac{1}{\log|x|}$ is discontinuous is ________.

37. If $f(x)=\begin{cases}ax+1, & if \quad x\geq 1\\ x+2, & if \quad x<1\end{cases}$ is continuous, then a should be equal to _______.

38. The derivative of $\log_{10}x$ w.r.t. x is ________.

39. If y = $\sec^{-1}\left(\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ + $\sin^{-1}\left(\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\dfrac{dy}{dx}$ is equal to ______.

40. The deriative of sin x w.r.t. cos x is ________.

41. The function f (x) = $\dfrac{x+1}{1+\sqrt{1+x}}$ is continuous at x = 0 if f (0) is _________.

42. An example of a function which is continuous everywhere but fails to be differentiable exactly at two points is __________ .

43. Derivative of x^2 w.r.t. x^3 is _________.

44. If f (x) = |cosx|, then $f'\left(\dfrac{\pi}{4}\right)$ = _______ .

45. If f (x) = |cosx – sinx | , then $f'\left(\dfrac{\pi}{3}\right)$ = _______.

46. For the curve $\sqrt{x}+\sqrt{y}=1$, $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{4},\dfrac{1}{4}\right)$ is __________.